

GENERAL LIBRARY
VOLUME LI
APR 27 1920
UNIV. OF MICH.

NUMBER 2

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE

Mount Wilson Observatory of the Carnegie
Institution of Washington

EDWIN B. FROST

Yerkes Observatory of the
University of Chicago

HENRY G. GALE

Ryerson Physical Laboratory of the
University of Chicago

MARCH 1920

ANNIBALE RICCO, 1644-1919	Giovanni Galli	65
DIFFRACTION OF A TELESCOPIC OBJECTIVE IN THE CASE OF A CIRCULAR SOURCE OF LIGHT	H. Nagels	73
ON COMET 1919 AND ON THE REJECTION OF A COMET'S TAIL	E. E. Barnard	102
PRELIMINARY OBSERVATIONS OF THE ZEEMAN EFFECT FOR ELECTRIC FURNACE SPECTRA	Arthur S. King	107
THE SPECTRUM OF NOVA OPHIUCHI 1919	Walter S. Adams and Cora G. Bancell	121
REVIEWS:		
<i>Theorie der Strahlung und der Quanten</i> , ARTHUR MARCH (A. C. Lunn), 127; <i>Advanced Lecture Notes on Light</i> , J. R. ECCLES (F.), 128.		
NOTE		128

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sondai
THE MISSION BOOK COMPANY, Shanghai

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE

Mount Wilson Observatory of the Carnegie
Institution of Washington

EDWIN B. FROST

Yerkes Observatory of the
University of Chicago

HENRY G. GALE

Ryerson Physical Laboratory of the
University of Chicago

WITH THE COLLABORATION OF

JOSEPH S. AMES, Johns Hopkins University

ARISTARCH BELOPOLSKY, Observatoire de Pulkova

WILLIAM W. CAMPBELL, Lick Observatory

HENRY CREW, Northwestern University

CHARLES FABRY, Université de Marseille

ALFRED FOWLER, Imperial College, London

CHARLES S. HASTINGS, Yale University

HEINRICH KAYSER, Universität Bonn

ALBERT A. MICHELSON, University of Chicago

HUGH F. NEWALL, Cambridge University

ERNEST F. NICHOLS, Yale University

ALFRED PEROT, Paris

CARL RUNGE, Universität Göttingen

HENRY N. RUSSELL, Princeton University

SIR ARTHUR SCHUSTER, Twyford

FRANK SCHLESINGER, Allegheny Observatory

The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. The subscription price is \$5.00 a year; the price of single copies is 65 cents. Orders for service of less than a half-year will be charged at the single-copy rate. Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Shanghai. Postage is charged extra as follows: For Canada, 30 cents on annual subscriptions (total \$5.30), on single copies, 3 cents (total 68 cents); for all other countries in the Postal Union, 62 cents on annual subscriptions (total \$5.62), on single copies, 11 cents (total 76 cents). Patrons are requested to make all remittances payable to The University of Chicago Press in postal or express money orders or bank drafts.

The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, 26s. each; single copies, including postage, 3s. 9d. each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 11.25 each; single copies, including postage, Yen 1.50 each.

For China: The Mission Book Company, 13 North Szechuan Road, Shanghai. Yearly subscriptions, \$5.00; single copies, 65 cents, or their equivalents in Chinese money. Postage extra, if mailed direct outside of Shanghai, on yearly subscriptions 62 cents, on single copies 11 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the Readers' Guide Supplement, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on
July 15, 1918.





ANNIBALE RICCÒ

1844-1919

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME LI

MARCH 1920

NUMBER 2

ANNIBALE RICCÒ, 1844-1919

BY GIORGIO ABETTI

Annibale Riccò was born in Modena on September 14, 1844. He took his degree in mathematics at the University of Modena in the year 1866 and in civil engineering at the *Politecnico* of Milan in 1868; in the same year also the degree of doctor in natural science at the University of Modena.

He then began his scientific career as assistant in the small observatory of his birthplace (1868-77) and during this period took part in the International Meteorological Conference in Vienna (1873), visiting then the principal observatories in Austria and Germany. In 1878 he was called as professor of Technological Physics in the *Scuola superiore di applicazione* in Naples, but this was not the branch of science for which he had so strong an inclination and the next year he accepted the position of astronomer in the *Specola* of Palermo, Cacciatore being director. In 1890 Riccò was elected director of the two new observatories of Catania and Mount Etna (at 2943 meters' elevation) and of the geodynamical service of Sicily and surrounding islands. There especially he developed his enormous scientific activity, being also director of the *Società degli Spettroscopisti Italiani* and editor of the *Memorie*. He took part in three eclipse expeditions and in many conferences and meetings for astrophysics and geodynamics. Being in Rome

for the *Comitato antisismico* which had to decide on the construction of antiseismical buildings in the regions of Southern Italy more shaken by earthquakes, he suffered a violent attack of malaria. He was transferred to the *Policlinico* of that city and after a few days he died quietly on September 23, 1919, just when he was to retire from active service.

In these days when every branch of science covers so large a field that one can hardly keep up with the development of one's specialty, it is difficult to understand how Riccò was able to pursue such different and wide branches of science. But this was in his time almost a necessity, for lack of men and institutions, and was a consequence of the particular location in which an active and indefatigable energy must always be ready to follow the mysterious convulsions of nature.

In the *Memorie della Società degli Spettroscopisti Italiani*, the well-known journal of the society founded by Secchi and Tacchini in 1872, principally for the observations and study of the solar phenomena, we find the first signs of his scientific activity in the year 1875, with a note: "On the succession and persistence of the sensations of the colors." From that time we have an uninterrupted record of the activity of Riccò in these *Memorie* which, edited first by P. Tacchini in Palermo, then in 1879 in Rome, were transferred in 1899 to Catania under the direction of Riccò, who gave to them his most complete and careful attention.

The first regular series of solar observations, direct and spectroscopic, made by him at the Royal Observatory at Palermo with the Merz refractor of 25 cm aperture and 442 cm focal length and with a direct vision Tauber spectroscope with ten prisms, are printed in the *Memorie* for the year 1880, and from that time until a few days before his death, when he left Catania for his last journey to Rome, the series is uninterrupted, so that we have complete statistics and record of the activity of the sun during thirty-eight years. As a matter of fact, he observed the sun the very morning of his departure for Rome, and to friends who were visiting him at the hospital he was explaining how on that morning he had observed prominences of a peculiar filamentous appearance such as he "had never seen before." These were almost his last conscious words.

In 1804 the brothers Gemellaro had erected, for the study of the crater of Mount Etna, near the top of the volcano, a small refuge where, in conjunction with a station at the foot of the mountain, systematic meteorological and volcanologic observations were carried on from that date. It was not until after the establishment of the kingdom of Italy that, due to the interest of Tacchini, it was possible to begin, in 1879, the construction of the *Osservatorio Etneo*, enlarging the house of the Gemellaro; and shortly after, the observatory in the town was also established. In 1886 Tacchini, showing the perfect conditions of the sky in Catania, obtained permission for the observatory to take part in the international work of the *Carte photographique du Ciel* for the zone $+47^{\circ}$ to $+54^{\circ}$. Since Tacchini, after the death of Father Secchi, had gone to the directorship of the observatory at the Collegio Romano, all the organization had to be done by Riccò who, with relatively small means, then had to provide for the installation of the instrument and for the work of observation, measurement, and reduction. These are well advanced, and some of the volumes of the astrographic catalogue containing the equatorial co-ordinates of the stars of that zone have been published by him and his collaborators.

But most of Riccò's attention was always given to the sun, and in this branch of astrophysics we find the greatest part of his work. Particular study was made by him of the relations between the sun-spots and the perturbations of terrestrial magnetism. He proves, with a number of observations, a retardation of the magnetic perturbation on the earth of 45^{h} after the passage of the sun-spot across the central meridian, which would mean a velocity of propagation of the magnetic disturbance of about one thousand kilometers per second.

Attempts to photograph the solar corona without an eclipse were made by Riccò and Dr. G. E. Hale on Mount Etna in the summer of 1894. The collaboration of the two scientists is most interesting, although the experiments were not successful. Mr. Hale had decided on the expedition during Tacchini's visit to Chicago in August, 1893, hoping that the conditions of the sky on Mount Etna, and the opportunity to use there the 12-inch equatorial, would give him a better chance than on his first expedition to Pike's

Peak. But during his stay at Mount Etna in July of that year, as Riccò writes, there was "unfortunately just at this time a certain increase in the activity of the central crater which was probably the prelude of the disastrous earthquakes on the eastern slope of the volcano which occurred on the 7th and 8th of August. The volume of smoke rising from the great crater was carried by the prevailing northwest wind over the observatory." The observations after the departure of Dr. Hale were continued by Riccò, who proved that, except at an eclipse, it was not possible to photograph any part of the corona.

Long after, in 1910, Riccò was able to obtain funds, to which the William E. Hale Fund contributed a notable part for the construction of a spectroheliograph to be attached to the Merz refractor of 30 cm aperture and 557 cm focal length, so that, from that time, daily spectroheliograms of the sun were a part of the regular observations made at Catania.

Special consideration was given to the statistics of the prominences and the study of their structure in the period 1880 to 1912, and the results presented by Riccò at the Fifth Conference of the International Solar Union establish important conclusions, as, for instance, the progressive decrease of the production of the prominences in the successive cycles of the activity of the sun and that the duration of the cycles of the prominences is approximately equal to that of the spots. There are two distinct kinds of prominences: those on the zone of the spots, very active, very variable, with hydrogen, helium, calcium, and other metals, on the one hand; and those of hydrogen only, quiescent, which appear in latitudes increasing from a maximum to the following one. Professor Schwarzschild, at the close of the report of Riccò, said: "I want to point out again the important result of Professor Riccò that the height in the frequency-curve of the sun-prominences is decreasing steadily. This means that also in the prominences there is an indication of great super-periods of the activity of the sun-spots. Such a result can be reached only with a continuous observation during a whole lifetime, as Professor Riccò has done."

The study of the prominences and dark filaments observed with the spectroheliograph on the surface of the sun brings Riccò to

this conclusion: "There cannot be any more doubt that the filaments and dark flocculi are due to absorption and obscuration of the light of the photosphere due to the prominences which exist on the solar disk at the moment of the spectroheliographic observation." This was suspected also by Hale and Ellerman from their first spectroheliograms taken at the Yerkes Observatory in 1903 and lately confirmed again with a splendid series of pictures taken in June, 1917, with the 13-foot spectroheliograph at Mount Wilson by Ellerman, which shows the dark patches carried by the sun's rotation to the limb and becoming prominences.

Riccò took part in three eclipse expeditions, in Algeria for that of May 28, 1900, at Alcalà de Chivert (Spain), August 30, 1905, and at Teodosia (Crimea), August 21, 1914. For the last two expeditions that he directed we find especially interesting reports in the *Memorie*. In Spain he observed the white prominences first discovered by Tacchini in 1883 and he noted then that "probably they are objects of intermediate nature between the prominences and the coronal streamers; that is, they are like a line of conjunction between these two classes of phenomena."

In the eclipse observed in Russia he did not find any white prominences, but in some of the prominences the "prevailing radiations of calcium because of their faint violet coloration give images white or almost white."

In the eclipse of 1905, when the solar activity was at its maximum, he obtained good photographs with the prismatic camera of the flash spectrum with the green coronal line λ 5303; in that of 1914, the sun being almost at the minimum of activity, he photographed, always with the prismatic camera, a new red band, confirmed by the observations of the other expeditions to the same eclipse, at λ 6374, which does not correspond to any known substance; he did not find any trace of the green coronal line.

Numerous observations were made by Riccò on the comets and their spectra, especially of the comets Morehouse and Halley. These observations gave him the occasion to discuss the hypothesis expressed by Professor Righi in his lecture on the physical constitution of the comets, which reaches the conclusion that we

have in them electric and optical phenomena analogous to those produced in the physical laboratories with the discharge tubes containing highly rarefied gases; analogous conditions to those of the comets which are constituted of matter extremely rarefied, wandering in the vacuum of sidereal space.

In the field of geodesy and geophysics Riccò made important observations for the determination of the relative gravity in Sicily, dividing the work with the geodesist, Professor Venturi, so that with their combined work they were able to make a chart of the anomalies of gravity in these interesting regions. Riccò determined the gravity with pendulum observations in 43 places in eastern Sicily, thus establishing relations between the anomalies of the terrestrial magnetism and those of the gravity and the seismic activity of those regions. Besides that, he was always a most intelligent observer of the phenomena of the volcanoes, and notable is his contribution to the researches on the eruption of Mount Etna in 1910, with a study of the central crater from 1892 to 1910.

After the death of Tacchini in 1906, he was invited to become a collaborator of this *Journal*, and was always interested in the intercourse between this paper and the *Memorie degli Spettroscopisti* and between the American and Italian scientists. He was one of the most active members of the International Union for Co-operation in Solar Research, and took part in the fourth conference held at Mount Wilson in 1910. After this visit he made a long report of the great development of astrophysics in the United States, describing the equipment and the lines of research followed in the American observatories, especially at Mount Wilson. He was present, too, at the fifth conference of the Union at Bonn in 1913, when he extended the invitation of the Italian government, accepted unanimously, that the next conference should be held in 1916 in Rome. This meeting, deferred by the obvious circumstances, will be held there in the near future, but the Union will regret the absence of one of its most distinguished members.

Considering all the various and indefatigable activity of Riccò one can see that his faculties of organization have been most successful in establishing the study of astrophysics and continuing

the fine traditions of Secchi, Donati, and Tacchini, and this is appreciated the more by those who know the limited means that he had at his disposal. Summing up the work done by him and his collaborators at the observatory of Catania, whether in establishing the observatory, or in its scientific production, it is really astonishing how much could be done with such small means, always obtained by hard efforts and with difficulty.

The scientific production of Riccò is collected chiefly in the *Memorie degli Spettroscopisti*, also in the *Comptes Rendus* of the French Academy, in the *Memorie* and *Atti* of the *Accademia dei Lincei*, of which he was elected national member in 1911, in the *Atti* of the *Accademia Gioenia*, of which he was elected president in 1899, in the *Rivista di Astronomia*, the *Astrophysical Journal*, and for geodynamics and meteorology in the Annals of the Italian weather bureau.

He received many Italian and foreign honors and prizes. In 1910 the *Accademia dei Lincei* awarded him the royal prize for astronomy. He was a member of numerous national and foreign academies and took part in many international meetings; the last one was that held at Brussels in July of this year under the auspices of the International Research Council, where the new International Astronomical Union was established. He was elected president of the section of volcanology.

He was for two years (1898-1900) rector of the Royal University of Catania and for eight years dean of the faculty of science at the same university. The gold medal for astrophysics of the French Academy and the Janssen medal were awarded to him, and he was Knight of the Crown of Italy and of S.S. Maurizio and Lazzaro.

The two events which marked the close of the scientific life of Riccò happened shortly before his death. The faculty of science of the University of Catania met on May 5, 1919, to deliberate upon the report of Riccò and others of his colleagues on the new Volcanologic Institute for Mount Etna, planned since 1910, which had to provide a chair for these studies separate from the one for astrophysics. He understood well that the two branches had so grown in a relatively small number of years that

independent institutes and workers were necessary for the development of each. The other event was the close of his official career and of his lectures at the university in July, 1919, when, having reached his seventy-fifth year, according to the Italian law, he had to retire. All his colleagues and students brought to him on that occasion their affectionate manifestations of esteem.

Always in good health and of great physical strength, he bore until his last days the fatigue of studies and of his many duties. He had a beloved and numerous family to whom he gave all his care and devoted affection. To science and his family he dedicated all of his life,

cui labore et fraudo animam meam bonis?—Ecclesiaste iv. 8.

He knew well how and for what purpose he had to give all his wealth of mind and labor.

As a tragic circumstance, his wife, succumbing to the same illness during the same days in Catania, two days after followed him to the better life. They both live and will live in the memory of their sons and of all their numerous and sincere friends.

ROME
December 1919

DIFFRACTION OF A TELESCOPIC OBJECTIVE IN THE CASE OF A CIRCULAR SOURCE OF LIGHT

By H. NAGAOKA

ABSTRACT

Diffraction of a telescope objective; distribution of intensity in the image of a circular object and of a combination of circular objects.—Many astronomical observations involve the question of the effect of diffraction on the images of circular objects. This problem is exhaustively treated in this paper. After deriving the general expression for the intensity of any part of the image of a circular source, in the form of an integral of a combination of cylinder functions and elliptic integrals, the author evaluates it for various parts of the image: the center, the periphery, and points just inside and outside the periphery. The results obtained for the intensity near the periphery, called the marginal intensity, are of particular interest. They enable diagrams to be constructed which show the isophotes for various stages of the following phenomena: (1) the transit of a bright point over a bright disk (Fig. 5); (2) the transit of one bright disk over another (Fig. 6); (3) the transit of a dark disk over a larger bright one (Fig. 7). The isophotes of a *luminous lune* are also given (Fig. 8).

Telescopic observation of transits; effect of diffraction phenomena.—The diagrams just mentioned show (1) that a luminous point approaching a luminous disk appears to enter the disk before it really is in line with the periphery, and (2) that the moment of contact of two disks is difficult to judge on account of the gradual transition of the isophotes. In the case of such occultation and transit phenomena, therefore, the time observations may vary with the instrument used.

In studying the diffraction phenomena produced by different forms of apertures, we generally assume a point source of light. This is, however, a rough approximation from a practical point of view; as regards the resolving power and the explanation of some astronomical observations, we have to assume a finite source of light. The diffraction of a circular aperture due to a point source was treated by Airy¹ and afterward perfected by Lommel.² For a finite source of light, the discussion was given by Rayleigh,³ André,⁴ H. Struve,⁵ Strehl,⁶ and Lommel.⁷ In a former paper,⁸ I was led to

¹ *Trans. Phil. Soc. (Cambridge)*, 6, 1838.

² *Abhand. Bayer. Akad.*, 15, 1884.

³ "Wave Theory of Light," *Encyc. Brit.*, 9th edition.

⁴ *Ann. l'Ecole Norm. Sup.*, 5, 1876; 10, 1881.

⁵ *Mem. l'Acad. St. Petersbourg*, 30, 1882.

⁶ *Theorie des Fernröhrs* (Leipzig), 1894.

⁷ *Abhand. Bayer. Akad.*, 19, 1897.

⁸ *Jour. Coll. Sci. (Tokyo)*, 9, 1898; *Phil. Mag.*, January 1898.

an approximate solution of the problem for a circular source of light, and obtained results which account for the drop formation during the ingress and the egress of the planets over the sun's disk. In the present discussion most of the difficulties hitherto encountered are overcome, and the result is made to apply to the practical solutions of problems concerning the distribution of light over a luminous disk as observed by means of a telescope. The result here obtained can be extended to the discussion of problems relating to the distribution of light, when bright disks are nearly in contact, or when a dark disk passes over a bright disk, and to questions of similar nature.

General formula.—In studying the diffraction due to a finite source of light, we start from the supposition that the independent sources of light do not give rise to interference effects. Dividing the source into elements, each of them sends out coherent waves which produce interference effects, but waves from any two of them do not interfere. The observed effect is therefore the sum of separate effects due to different elements of the source. A diffraction pattern due to a point source, such as a fixed star, produced by a circular aperture, consists of a system of concentric rings; when the source is of a finite area, the intensity of light in the focal plane of the observing telescope is an integral effect due to all the elements of the source. The problem is to find the intensity of the image of a uniform luminous disk, and then to proceed to find the effect of superposing luminous and dark disks, and combinations of similar nature.

The problem to be discussed naturally belongs to Fraunhofer's diffraction phenomena, as the source is placed at an infinite distance and the telescope is focused to it. Taking the plane of the aperture of the telescope for the xy -plane, denote the cosines of the angles which the incident ray makes with the x - and y -axes by α and β and those for the diffracted ray by α' and β' ; then putting

$$r = \frac{2\pi\sqrt{(\alpha - \alpha')^2 + (\beta - \beta')^2}}{\lambda} R,$$

where R denotes the radius of the objective, and λ the wave-length of light, the intensity of the diffracted ray in the focal plane of the telescope is proportional to

$$\frac{J_1^2(r)}{r^2},$$

where $J_1(r)$ denotes the cylinder function of the first kind and of order 1, with argument r . For normal incidence $\alpha = \beta = 0$, and

$$r = \frac{2\pi R \sin \phi}{\lambda},$$

where ϕ is the angle made by the diffracted ray with the axis of the telescope. For $R = 1$ cm, $\lambda = 0.5 \mu$, $\phi = 1'$, the value of $r = 36.55$, so that for most of the telescopes r becomes tolerably large even for small angles of diffraction; in practical application the limits for r lie between 0 and a large value, which will be denoted by a . For a finite source of light we have to consider α and β as variables, corresponding to the diffracted ray in direction α' , β' , and integrate the effects due to all elements of the luminous disk. Thus the intensity is proportional to

$$\int \frac{J_1^2(r)}{r^2} d\sigma, \quad (1)$$

where $d\sigma$ is a surface element of the source. Expressed in polar co-ordinates r , θ , the above integral becomes

$$c \int \int \frac{J_1^2(r)}{r} dr d\theta.$$

To fix the factor of proportionality, c , we shall assume the intensity due to an infinite disk of uniform brightness to be unity. Since

$$\frac{J_1^2(r)}{r} = -\frac{1}{2} \frac{d}{dr} (J_0^2(r) + J_1^2(r))$$

$$\int \int \frac{J_1^2(r)}{r} d\theta dr = \frac{1}{2} \int (1 - J_0^2(r) - J_1^2(r)) d\theta,$$

the value of the integral at the center of a circular disk of infinite radius and of uniform brightness

$$\int_0^{2\pi} \int_0^{\infty} \frac{J_0^2(r)}{r} d\theta dr = \pi.$$

Consequently, the factor of proportionality is $\frac{I}{\pi}$ and the intensity

$$I = \frac{I}{2\pi} \int (1 - J_0^2(r) - J_1^2(r)) d\theta. \quad (2)$$

To find the intensity of light at any point P in the plane of the disk, let $AP = r$, $OA = a$, $OP = \nu a$, where $OP/OA = \nu < 1$, as P lies inside or outside the disk. Then on account of the geometrical condition

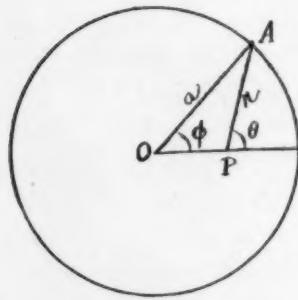


FIG. 1

$$r^2 = a^2(1 - 2\nu \cos \phi + \nu^2)$$

and

$$a \cos \theta = \nu a + r \cos \theta,$$

we have

$$d\theta = \left(\frac{1 - \nu \cos \phi}{1 - 2\nu \cos \phi + \nu^2} \right) d\phi;$$

consequently the expression for the intensity becomes

$$I = \frac{I}{2\pi} \int_0^{2\pi} [1 - J_0^2(r) - J_1^2(r)] \frac{1 - \nu \cos \phi}{1 - 2\nu \cos \phi + \nu^2} \cdot d\phi. \quad (3)$$

Putting

$$\phi = \pi - 2\psi, \quad \cos \phi = 2 \sin^2 \psi - 1,$$

we have

$$r = a(1 + \nu) \sqrt{1 - k^2 \sin^2 \psi} = a \sqrt{1 - k^2 \sin^2 \psi}, \quad (4)$$

where

$$k^2 = \frac{4\nu}{(1+\nu)^2}, \quad \text{and } a = a(1+\nu),$$

and

$$1 - k^2 = k'^2 = \left(\frac{1-\nu}{1+\nu}\right)^2.$$

For points inside the disk

$$k' = \frac{1-\nu}{1+\nu}, \quad (5)$$

and outside it

$$k' = \frac{\nu-1}{\nu+1}. \quad (5')$$

For points near the margin, ν is nearly equal to 1, and we shall put

$$\nu = 1 \mp \epsilon, \quad (5'')$$

where ϵ is generally a small quantity. We remark that for marginal points

$$k' = \frac{\epsilon}{2} \quad (5''')$$

and

$$k = 1$$

nearly, neglecting small terms of higher order.

Further

$$\frac{1 - \nu \cos \phi}{1 - 2\nu \cos \phi + \nu^2} = \frac{1}{2} \left(1 \pm \frac{k'}{1 - k^2 \sin^2 \psi} \right) \quad (6)$$

where the + sign is to be taken for internal, and the - sign for external, points.

For simplifying the expression, put

$$u = \int_0^\psi \frac{d\psi}{\sqrt{1 - k^2 \sin^2 \psi}}$$

and $u = K$ for $\psi = \frac{\pi}{2}$, K denoting a complete elliptic integral of the first kind. Borrowing the notation of the elliptic functions

$$\sqrt{1 - k^2 \sin^2 \psi} = \operatorname{dn} u$$

and

$$\frac{k'}{\operatorname{dn} u} = (\operatorname{dn} u + K),$$

we obtain for (4) and (6)

$$r = a(1 + \nu) \operatorname{dn} u = a \operatorname{dn} u \quad (4')$$

$$\frac{1 - \nu \cos \phi}{1 - 2\nu \cos \phi + \nu^2} d\psi = \frac{1}{2} [\operatorname{dn} u \pm \operatorname{dn}(u + K)] du, \quad (6')$$

and the expression for the intensity (3) becomes

$$I = \frac{1}{\pi} \int_0^K \{1 - J_o^2(a \operatorname{dn} u) - J_i^2(a \operatorname{dn} u)\} [\operatorname{dn} u \pm \operatorname{dn}(u + K)] du. \quad (7)$$

Thus the present problem is reduced to the evaluation of I in (7), and the proper interpretation of the result thus obtained.

It is convenient to discuss the result in four steps: (1) the intensity at the center of the disk; (2) the intensity at the periphery of the disk; (3) the intensity at points internal and external to the disk; (4) the intensity near the margin.

Intensity at the center of the disk.—For finding the intensity at the center of the disk, we have to put $\nu = 0$, whence $k = 0$, and $\operatorname{dn} u = 1$,

$$\operatorname{dn}(u + K) = 1, \quad a = a, \quad \text{and} \quad K = \frac{\pi}{2}.$$

Thus

$$I_0 = 1 - J_o^2(a) - J_i^2(a). \quad (8)$$

The above value can be found directly from (2) without having recourse to (7).

Since J_o and J_i are fluctuating functions, $J_o^2 + J_i^2$ have several singular characteristics, which were discussed in my former paper.

Plotting the curve

$$y = J_o^*(x) + J_i^*(x) \quad (9)$$

we find that it shows a succession of steps at nearly equal horizontal intervals (Fig. 2a); the height of the consecutive steps becomes smaller and the rate of decrease diminishes as we recede from the axis of y . The mean curve for values of x greater than the first root $x_1 = 3.8317$ of $J_i(x)$ is approximately a rectangular hyperbola

$$xy = \frac{2}{\pi}. \quad (9')$$

This comes from the following property of the function. y defined by (9) is less than 1, for all values of x , except for $x=0$; it gradually diminishes with increasing values of x . Points corresponding to the roots of $J_i(x)=0$ are points of inflexion and have tangents parallel to the x -axis; these points occur at nearly equal intervals of the abscissae little greater than π , for values of x greater than the first root x_1 ; the consequence is that the curve has neither maximum nor minimum, excepting the point $x=0, y=1$, as shown in Fig. 2b.

To calculate the numerical value of the intensity at the center, we have to evaluate $J_o^*(a) + J_i^*(a)$. For small values of a , we find by expanding

$$J_o^*(a) + J_i^*(a) = \frac{2}{\pi} \int_0^\pi J_o(2a \sin \omega) \cos^2 \omega d\omega$$

in power series,

$$J_o^*(a) + J_i^*(a) = \sum_o \frac{(-1)^n \Pi(2n)}{2^{2n} (\Pi n)^3 \Pi(n+1)} a^{2n} = \sum_o A_n a^{2n} \quad (10)$$

$$\text{where } A_0 = 1; A_1 = \frac{1}{2}; A_2 = \frac{5}{36}; A_3 = \frac{7}{288}; A_4 = \frac{7}{2400}; A_5 = \frac{11}{43200};$$

$$A_6 = \frac{143}{8467200}; A_7 = \frac{2 \cdot 5 \cdot 11 \cdot 13}{(8!)^2}; A_8 = \frac{2 \cdot 11 \cdot 13 \cdot 17}{(9!)^2}; A_9 = \frac{4 \cdot 13 \cdot 17 \cdot 19}{(10!)^2};$$

$$A_{10} = \frac{2 \cdot 7 \cdot 13 \cdot 17 \cdot 19}{(11!)^2}. \text{ For large values of } a, \text{ the semi-convergent expansion}$$

$$J_o^*(a) + J_i^*(a) = \frac{2}{\pi a} \left(1 + \frac{1}{8a^2} - \frac{\cos 2a}{2a} - \frac{\sin 2a}{8a^2} + \dots \right) \quad (11)$$

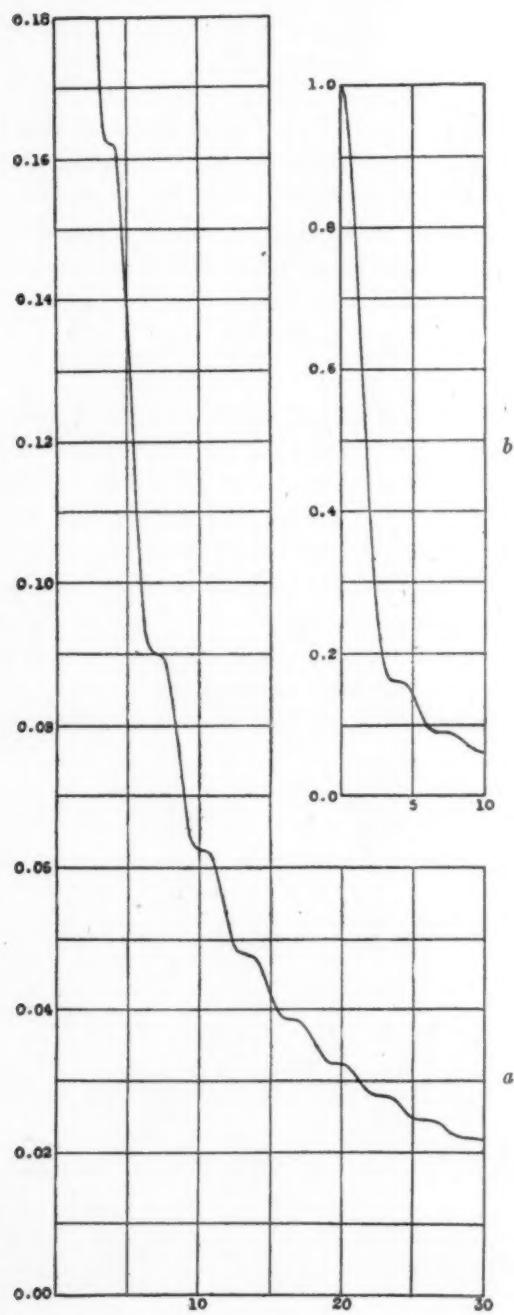


FIG. 2

is rapidly convergent, and can be conveniently used for values of a greater than the first root x_1 of $J_1(x)=0$; for this value of a , the above expression is accurate to the fourth decimal place. (9') follows at once from (11). Other expressions for y can be deduced, but they are simply of mathematical interest, so that they will be omitted in the present paper.

As to the intensity at the center of the disk, we remark that by dividing the disk into a series of zones bounded by circles whose radii are equal to the roots of $J_1(a)=0$, we find for the first circle ($a=3.8317$) $I=0.83778$; for the second circle ($a=7.0156$) $I=0.90994$; for the third circle ($a=10.1735$) $I=0.93765$; and so on at a decreasing rate. The difference in intensity between successive circles gradually diminishes with the increasing value of a , and the intensity at the center of a large disk is given by the approximate expression

$$I_0 = 1 - \frac{2}{\pi a}, \quad (12)$$

which is the natural consequence of equation (11). As already noticed, a is generally large in practical applications, so that (12) is useful for comparing the intensity of a bright disk with that of an infinite extent. The numerical values are given in Table I later on; in it will be found the values of $1 - I_0$, which is nearly equal to $\frac{2}{\pi a}$. The conclusion is that the intensity at the center of a circular disk is very little smaller than that of an infinite plate of the same intrinsic brightness.

Intensity at the periphery.—The intensity at the periphery of the disk can be found by putting $v=1$ in (7). This case corresponds to $k=1$, $k'=0$, $dn u = \cos \psi$, $dn(u+K)=0$, and $a=2a$; thus $r=2a \cos \psi$ and

$$\begin{aligned} I_p &= \frac{1}{\pi} \int_0^{2a} [1 - J_0^2(r) - J_1^2(r)] \frac{dr}{\sqrt{4a^2 - r^2}} \\ &= \frac{1}{2} - \frac{1}{\pi} \int_0^{2a} [J_0^2(r) + J_1^2(r)] \frac{dr}{\sqrt{4a^2 - r^2}}. \end{aligned} \quad (13)$$

As we assume a to be tolerably large, the integral is to be divided into two parts, the limits of the first integral being from 0 to $x_1 = 3.8317$, which is the first root of $J_1(x) = 0$; in this integral $J_0^2 + J_1^2$ is to be expanded in power series. In the second integral, in which the limits lie between x and $2a$, we may conveniently employ the semi-convergent series (11) for $J_0^2 + J_1^2$ and effect the

TABLE I

a	I_0	$I - I_0$	$\frac{1}{2}I_p$	I_p
20.....	0.9676	0.0324	0.0306	0.4694
25.....	0.9750	0.0250	0.0254	0.4746
30.....	0.9784	0.0216	0.0218	0.4782
35.....	0.9820	0.0180	0.0191	0.4809
40.....	0.9841	0.0150	0.0171	0.4829
45.....	0.9858	0.0142	0.0154	0.4846
50.....	0.9874	0.0126	0.0141	0.4859
60.....	0.9895	0.0105	0.0121	0.4879
70.....	0.9909	0.0091	0.0106	0.4894
80.....	0.9920	0.0080	0.0094	0.4906
90.....	0.9929	0.0071	0.0085	0.4915
100.....	0.9936	0.0064	0.0078	0.4922
150.....	0.9958	0.0042	0.0055	0.4945
200.....	0.9968	0.0032	0.0042	0.4958
250.....	0.9975	0.0025	0.0035	0.4965
300.....	0.9979	0.0021	0.0030	0.4970
400.....	0.9984	0.0016	0.0023	0.4977
500.....	0.9987	0.0013	0.0019	0.4981
600.....	0.9989	0.0011	0.0016	0.4984
700.....	0.9991	0.0009	0.0014	0.4986
800.....	0.9992	0.0008	0.0012	0.4988
900.....	0.9993	0.0007	0.0011	0.4989
1000.....	0.9994	0.0006	0.0010	0.4990
1500.....	0.9996	0.0004	0.0007	0.4993
2000.....	0.9997	0.0003	0.0005	0.4995
2500.....	0.9997	*0.0003	0.0004	0.4996
3000.....	0.9998	0.0002	0.0004	0.4996

integration. After a somewhat tedious process of expansion and integration, we find that for the first part of the integral

$$\frac{1}{\pi} \int_0^{x_1} [J_0^2(r) + J_1^2(r)] \frac{dr}{\sqrt{4a^2 - r^2}} = \frac{0.302117}{a} + \frac{0.093952}{a^3} + \frac{0.12142}{a^5} + \dots \quad (14a)$$

For the second part we use the expansion (11) in (13); thus the integral sought consists of the following parts:

$$\begin{aligned} \frac{2}{\pi^2} \int_{x_1}^{2a} \frac{dr}{r\sqrt{4a^2-r^2}} &= -\frac{1}{\pi^2 a} \log n \frac{x_1}{4a} - \frac{x_1^2}{16\pi^2 a^3} + \dots \\ &= \frac{0.004355}{a} + \frac{0.233301}{a} \log_{10} a - \frac{0.09297}{a^3} \end{aligned} \quad (14b)$$

$$\frac{1}{4\pi^2} \int_{x_1}^{2a} \frac{dr}{r^3\sqrt{4a^2-r^2}} = \frac{0.000431}{a} + \frac{0.003645}{a^3} \log_{10} a - \frac{0.00086}{a^3} + \dots \quad (14c)$$

The oscillating term gives

$$-\frac{1}{\pi^2} \int_{x_1}^{2a} \frac{\cos 2r dr}{r^2\sqrt{4a^2-r^2}} = \frac{0.0749}{\pi^3 a} + \dots = \frac{0.00242}{a} + \dots \quad (14d)$$

This last integral is rather difficult to evaluate, as x_1 is not a multiple of $\frac{\pi}{2}$; we have to evaluate first the integral from x_1 to $\frac{3\pi}{2}$, and then through successive intervals of π . The formula (14d) is only applicable when a is tolerably large. The integral coming from the term $\frac{\sin 2r}{4\pi r^3}$ in the expansion of $J_0^2 + J_1^2$ is negligibly small. Adding (14a, b, c, d), we find for the intensity at the periphery of a circular disk

$$I_p = \frac{1}{2} - \left(\frac{0.3093}{a} + \frac{0.2333}{a} \log_{10} a + \frac{0.0036}{a^3} \log_{10} a - \dots \right) \quad (15)$$

The expansion (15) shows that the peripheral intensity for a large disk is nearly half the intensity at the center. The following table gives the intensity at the center of different disks I_0 and that at the periphery I_p , and shows how the foregoing conclusion is a close approximation. Graphically represented, the curve takes the form given in Fig. 3 for $\epsilon = 0.000$.

Intensity inside and outside the disk.—For finding the value of the intensity at points neither near the center nor the margin of the disk, whose radius a is tolerably large, the integral can be reduced in the following manner.

Since a is large, and $dnu > 0$, we can expand $J_o^2 + J_i^2$ and retain the first term. Thus

$$J_o^2(a dnu) + J_i^2(a dnu) = \frac{2}{\pi a dnu} \text{ or } = \frac{2(dnu + K)}{\pi a k'}. \quad (16)$$

The integral (7) thus becomes

$$I = \frac{1}{\pi} \int_0^K \left(1 - \frac{2}{\pi a dnu} \right) [dnu \neq dn(u+K)] du. \quad (17)$$

Evaluating the integral, we find the intensity in terms of the complete elliptic integrals of the first and second kind, K and E respectively.

$$\begin{aligned} I_i &= 1 - \frac{1}{\pi^2 a} \left(\frac{E}{k'} + K \right) \\ &= 1 - \frac{2}{\pi^2 (1+\nu) a} \left(\frac{E}{k'} + K \right) \end{aligned} \quad (18)$$

for points *internal* to the disk.

$$I_e = \frac{2}{\pi^2 (1+\nu) a} \left(\frac{E}{k'} - K \right) \quad (18')$$

for points *external* to the disk. The expansion is correct to three or four decimal places for values of the argument $a dnu$ greater than the first root x_1 of $J_i(x) = 0$; thus for practical purposes the approximation here introduced is generally sufficient, as the fourth decimal is of little significance for the present problem.

For small values of k' we can conveniently utilize the approximate expressions for the elliptic integrals.

$$\begin{aligned} K &= \log \frac{4}{k'} + \frac{1}{4} k'^2 \left(\log \frac{4}{k'} - 1 \right) + \dots \\ \frac{E}{k'} &= \frac{1}{k'} + \frac{1}{2} k' \left(\log \frac{4}{k'} - \frac{1}{1.2} \right) + \dots \end{aligned} \quad (19)$$

Substituting in (18) and (18') and noticing that $\frac{1}{k'} = \frac{1+\nu}{1-\nu}$, we obtain for the intensities at points inside and outside the disk under the form

$$\begin{aligned} I_i &= 1 - \frac{2}{\pi^2(1+\nu)a} \left\{ \left(1 + \frac{k'}{2}\right) \log \frac{4}{k'} - \frac{k'}{4} \right\} - \frac{2}{\pi^2(1-\nu)a} \\ I_e &= \frac{2}{\pi^2(1+\nu)a} \left\{ \left(1 - \frac{k'}{2}\right) \log \frac{4}{k'} + \frac{k'}{4} \right\} - \frac{2}{\pi^2(1-\nu)a} \end{aligned} \quad (20)$$

In finding the intensity for values of k not near 1, we can easily utilize Landen's transformation. Putting

$$k_i = \frac{1-k'}{1+k'} = \nu \quad \dots \text{for internal point,}$$

$$= \frac{1}{\nu} \quad \dots \text{for external point;}$$

$$\begin{aligned} k_2 &= \frac{1-k'_i}{1+k'_i} = \left(\frac{\nu}{1+\sqrt{1-\nu^2}} \right)^2 \text{ for internal point,} \\ &= \left(\frac{1}{\nu+\sqrt{\nu^2-1}} \right)^2 \text{ for external point;} \end{aligned}$$

we know from the theory of elliptic functions that

$$K = \frac{\pi}{2}(1+k_1)(1+k_2) \quad \dots$$

$$E = \{(1-k)^2 + \frac{1}{2}k^2(1 - \frac{1}{2}k_1 - \frac{1}{4}k_1k_2 + \dots)\}K.$$

Consequently, the terms entering into the expressions for the intensity (18) and (18') become

$$\frac{E}{k'} + K = \left(\frac{1+k'}{2k'} \right)^2 \left\{ 1 - \frac{k_1^2}{2}(1 + \frac{1}{2}k_2) \right\} K \quad (21)$$

and

$$\frac{E}{k'} - K = \frac{(1-k')^2}{4k'} \left(1 - \frac{k_2}{2} \right) K \quad (21')$$

nearly. These substituted in (18) and (18') give the intensities not very near the margin.

Tables II and III give the values of $\frac{2}{\pi^2(1+\nu)} \left(\frac{E}{k'} + K \right)$ for finding I_i , and $\frac{2}{\pi^2(1+\nu)} \left(\frac{E}{k'} - K \right)$ for I_e , and the values of the constituent two terms multiplied by a for ν very near unity are also given. These can be used only for large values of a , such that $a\epsilon$ is less than x_1 .

TABLE II

	ϵ	$\frac{2}{\pi^2(1+\nu)} \left\{ \left(1 + \frac{k'}{2} \right) \log \frac{4}{k'} - \frac{k'}{4} \right\}$	$\frac{2}{\pi^2(1-\nu)}$	$\frac{2}{\pi^2(1+\nu)} \left(\frac{E}{k'} + K \right)$
0.990.....	0.010	0.682	20.264	20.946
0.991.....	0.009	0.692	22.516	23.208
0.992.....	0.008	0.704	25.330	26.034
0.993.....	0.007	0.717	28.949	29.666
0.994.....	0.006	0.732	33.774	34.506
0.995.....	0.005	0.750	40.528	41.478
0.996.....	0.004	0.772	50.661	51.433
0.997.....	0.003	0.801	67.547	68.348
0.998.....	0.002	0.841	101.321	102.162
0.999.....	0.001	0.911	202.642	203.553
0.9999.....	0.0001	1.144	2026.4	2027.6
	$-\epsilon$	$\frac{2}{\pi^2(1+\nu)} \left\{ \left(1 - \frac{k'}{2} \right) \log \frac{4}{k'} - \frac{k'}{4} \right\}$	$-\frac{2}{\pi^2(1-\nu)}$	$\frac{2}{\pi^2(1+\nu)} \left(\frac{E}{k'} - K \right)$
1.010.....	0.010	-0.673	20.264	19.591
1.009.....	0.009	-0.684	22.516	21.832
1.008.....	0.008	-0.696	25.330	24.034
1.007.....	0.007	-0.710	28.949	28.239
1.006.....	0.006	-0.726	33.774	33.048
1.005.....	0.005	-0.745	40.528	39.783
1.004.....	0.004	-0.768	50.661	49.893
1.003.....	0.003	-0.798	67.547	66.749
1.002.....	0.002	-0.839	101.321	100.482
1.001.....	0.001	-0.910	202.642	201.732
1.0001.....	0.0001	-1.144	2026.4	2025.3

From the tables it is evident that the intensity is very nearly equal to 1 over the surface of the disk; only when $\nu = 1 - \epsilon$ is near unity, or when ϵ is very small, is its variation considerable. Thus the intensity diminishes to the value $\frac{1}{2}$ near the margin of the disk. On passing the periphery to the outside ($\epsilon < 0$), the change in inten-

sity is quite rapid and tends toward zero. Instead of giving the values of the intensity for different ϵ , it is shown graphically in Fig. 3. The numerals under each curve give the values of ϵ , the radii a of the disk, calculated according to the formula already given, being indicated by the abscissae and the intensities by the ordinates. For a given radius of the disk, the intensities at different points inside and outside it are found by cutting the curve by a straight line parallel to the ordinate through the abscissa corresponding to a . Figure 4 shows how the intensity varies from inside to outside for different values of the radius a .

TABLE III

ν	ϵ	$\frac{2}{\pi^2(1+\nu)} \left(\frac{E}{k'} + K \right)$	ν	$-\epsilon$	$\frac{2}{\pi^2(1+\nu)} \left(\frac{E}{k'} - K \right)$
0.05.....	0.95	0.6378	1.01.....	0.01	19.591
0.10.....	0.90	0.6414	1.02.....	0.02	9.533
0.15.....	0.85	0.6476	1.03.....	0.03	6.199
0.20.....	0.80	0.6565	1.04.....	0.04	4.542
0.25.....	0.75	0.6683	1.05.....	0.05	3.554
0.30.....	0.70	0.6830	1.1.....	0.1	1.6077
0.35.....	0.65	0.7027	1.2.....	0.2	0.6779
0.40.....	0.60	0.7266	1.3.....	0.3	0.3898
0.45.....	0.55	0.7562	1.4.....	0.4	0.2560
0.50.....	0.50	0.7930	1.5.....	0.5	0.1813
0.55.....	0.45	0.8392	1.6.....	0.66	0.1349
0.60.....	0.40	0.8980	1.7.....	0.7	0.1040
0.65.....	0.35	0.9746	1.8.....	0.8	0.0824
0.70.....	0.30	1.0773	1.9.....	0.9	0.0667
0.75.....	0.25	1.2214	2.0.....	1.0	0.0549
0.80.....	0.20	1.4369	3.0.....	2.0	0.0135
0.85.....	0.15	1.7936	4.0.....	3.0	0.0053
0.90.....	0.10	2.4994	5.0.....	4.0	0.0027
0.95.....	0.05	4.5838	10.0.....	9.0	0.003
0.96.....	0.04	5.617	100.0.....	99.0	0.0000
0.97.....	0.03	7.332			
0.98.....	0.02	10.747			
0.99.....	0.01	20.947			

From Fig. 3 we see that for ϵ nearly equal to zero, the curves of intensity are much curved for small values of a . To obtain the exact value, we must have recourse to another method of expansion, which is semi-convergent and can be expressed in terms of a cylinder function of the first kind and of different orders. The expansion was given by Lommel in his theory of twilight, for which the diffraction aperture is of the order of a micron, representing the dimension

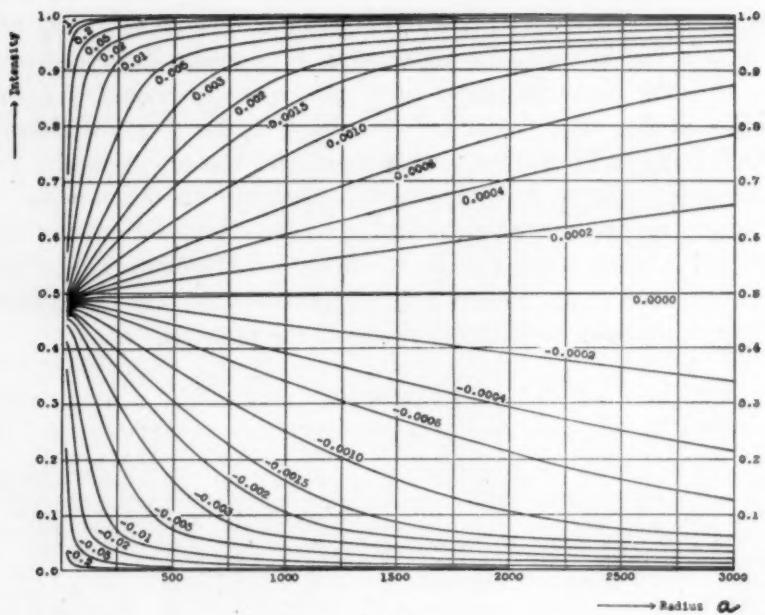


FIG. 3

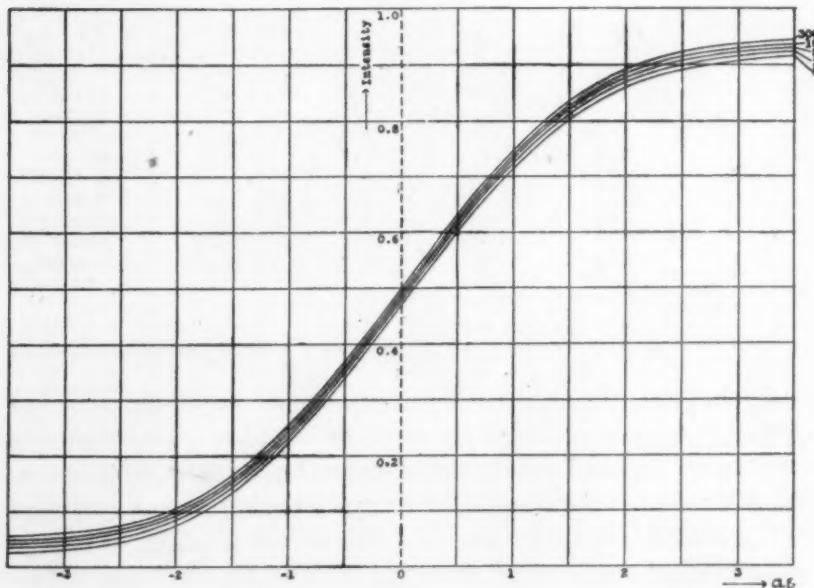


FIG. 4

of a dust particle. Such a case is never met with in telescopes and consequently the discussion is omitted in the present paper.

Marginal intensity.—As noticed above, the intensity near the periphery, which we shall call the marginal intensity, is of great interest in the diffraction phenomena of telescopic objectives. The expression is, however, most difficult to evaluate, and the numerical calculation somewhat tedious. In the papers hitherto published, several approximate expressions were obtained, sometimes expanded in series and sometimes evaluated by means of mechanical quadratures. In spite of these difficulties I found that the marginal intensity can be obtained by separating the integral giving the intensity into several parts, of which the principal consists of the peripheral intensity of a bright disk whose radius is somewhat smaller than that in question.

In the first place, it is to be remarked that in the general expression for the intensity

$$I = I_1 + I_2 = \frac{1}{\pi} \int_0^K \{ 1 - J_0^2(a dnu) - J_1^2(a dnu) \} [dnu \pm (dnu + K)] du \quad (7)$$

k is very nearly equal to 1 near the margin; consequently $dnu = \sqrt{1 - k^2 \sin^2 \psi} = \cos \psi$ to a close approximation. To take an example, for $\epsilon = 0.02$, $k' = 0.01$, and $k = 0.99995$.

The first part of the integral (7) thus reduces to

$$I_1 = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \{ 1 - J_0^2(a \cos \psi) - J_1^2(a \cos \psi) \} d\psi, \quad (21)$$

which is the value of the peripheral intensity of the disk of radius $\frac{a}{2} = \frac{a(1+\nu)}{2}$ and may be expressed by

$$I_1 = I_p \left(\frac{a}{2} \right) = I_p \left(\frac{a(1+\nu)}{2} \right). \quad (22)$$

As already noticed, I_p is nearly equal to $\frac{1}{2}$ when a is tolerably large and can be found from Table I; the deviation from the peripheral

intensity is given by the second term affected with \pm sign, where + refers to internal points and - to the external.

The second integral is equivalent to

$$I_2 = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left\{ 1 - J_0^2(a(1+\nu)\sqrt{1-k^2 \sin^2 \psi}) - J_1^2(a(1+\nu)\sqrt{1-k^2 \sin^2 \psi}) \right\} \frac{k' d\psi}{1-k^2 \sin^2 \psi} \quad (23)$$

We shall separate the integral into two parts, such that the limits lie between 0 to ϕ_1 with respect to ϕ , where $\phi = \frac{\pi}{2} - \psi$, and 0 to $\frac{\pi}{2} - \phi_1$ with respect to ψ , ϕ_1 being a small angle defined by the relation

$$a\sqrt{k'^2 + k^2\phi^2} = x_1, \quad (24)$$

where x_1 has the usual meaning. Then

$$\begin{aligned} & \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left\{ 1 - J_0^2(a\sqrt{1-k^2 \sin^2 \psi}) - J_1^2(a\sqrt{1-k^2 \sin^2 \psi}) \right\} \frac{k' d\psi}{1-k^2 \sin^2 \psi} \\ &= \frac{1}{\pi} \int_0^{\phi_1} \left\{ 1 - J_0^2(a\sqrt{k'^2 + k^2\phi^2}) - J_1^2(a\sqrt{k'^2 + k^2\phi^2}) \right\} \frac{k' d\phi}{k'^2 + k^2\phi^2} \quad (25) \\ &+ \frac{1}{\pi} \int_0^{\frac{\pi}{2}-\phi_1} \left\{ 1 - J_0^2(a\sqrt{1-k^2 \sin^2 \psi}) - J_1^2(a\sqrt{1-k^2 \sin^2 \psi}) \right\} \frac{k' d\psi}{1-k^2 \sin^2 \psi} \end{aligned}$$

The first integral can be easily expressed by putting

$$a\sqrt{k'^2 + k^2\phi^2} = x,$$

so that

$$xdx = a^2 k^2 \phi d\phi$$

and

$$\frac{d\phi}{k'^2 + k^2\phi^2} = \frac{adx}{kx\sqrt{x^2 - a^2 k'^2}}.$$

Consequently the integral becomes

$$\frac{ak'}{\pi} \int_{ak'}^{x_1} \{ 1 - J_0^2(x) - J_1^2(x) \} \frac{dx}{kx\sqrt{x^2 - a^2 k'^2}} = S. \quad (26)$$

Expanding the expression in parenthesis in powers of x , and integrating by parts, we obtain

$$S = \frac{k'a}{2\pi k} \sqrt{x_1^2 - k'^2 a^2} \left\{ 0.31845 - 0.47814 \left(\frac{k'a}{4} \right)^2 - 0.46770 \left(\frac{k'a}{4} \right)^4 \right. \\ \left. - 0.29870 \left(\frac{k'a}{4} \right)^6 + 0.13284 \left(\frac{k'a}{4} \right)^8 - 0.04339 \left(\frac{k'a}{4} \right)^{10} + \dots \right\}$$

Remembering that $a = 2a$ and $k' = \frac{\epsilon}{2}$, $k = 1$ nearly,

$$S = \frac{a\epsilon}{\pi} \sqrt{14.682 - a^2 \epsilon^2} \left\{ 0.15923 - 0.23907 \left(\frac{a\epsilon}{4} \right)^2 + 0.23385 \left(\frac{a\epsilon}{4} \right)^4 \right. \\ \left. - 0.14935 \left(\frac{a\epsilon}{4} \right)^6 - 0.06642 \left(\frac{a\epsilon}{4} \right)^8 - 0.0217 \left(\frac{a\epsilon}{4} \right)^{10} + 0.0054 \left(\frac{a\epsilon}{4} \right)^{12} \right. \\ \left. - 0.00107 \left(\frac{a\epsilon}{4} \right)^{14} + 0.0002 \left(\frac{a\epsilon}{4} \right)^{16} - \dots \right\} \quad (27)$$

S is given in Table IV for values of $a\epsilon$ from 0 to x_1 .

As to the second integral, we remark that the first term gives

$$\frac{k'}{\pi} \int_0^{\frac{\pi}{2} - \phi_1} \frac{d\psi}{1 - k^2 \sin^2 \psi} = \frac{k'}{\pi} \int_0^{\frac{\pi}{2}} \frac{d\psi}{1 - k^2 \sin^2 \psi} - \frac{k'}{\pi} \int_{\frac{\pi}{2} - \phi_1}^{\pi} \frac{d\psi}{1 - k^2 \sin^2 \psi} \\ = \frac{1}{2} - \frac{1}{\pi k} \cos^{-1} \frac{k'a}{x_1} \\ = \frac{1}{2} - \frac{1}{\pi} \cos^{-1} \frac{a\epsilon}{x_1} \quad (28)$$

nearly. The values of $\cos^{-1} \frac{a\epsilon}{x_1}$ are given in Table V.

TABLE IV

$\theta\epsilon$	S	$\theta\epsilon$	S
0.0.....	0.0000	2.0.....	0.2334
0.1.....	0.0194	2.1.....	0.2320
0.2.....	0.0387	2.2.....	0.2300
0.3.....	0.0576	2.3.....	0.2268
0.4.....	0.0761	2.4.....	0.2225
0.5.....	0.0941	2.5.....	0.2171
0.6.....	0.1113	2.6.....	0.2109
0.7.....	0.1276	2.7.....	0.2037
0.8.....	0.1432	2.8.....	0.1958
0.9.....	0.1576	2.9.....	0.1868
1.0.....	0.1709	3.0.....	0.1770
1.1.....	0.1830	3.1.....	0.1663
1.2.....	0.1939	3.2.....	0.1545
1.3.....	0.2034	3.3.....	0.1418
1.4.....	0.2117	3.4.....	0.1276
1.5.....	0.2185	3.5.....	0.1117
1.6.....	0.2241	3.6.....	0.0933
1.7.....	0.2284	3.7.....	0.0700
1.8.....	0.2311	3.8.....	0.0344
1.9.....	0.2327	3.8317.....	0.0000

TABLE V

$\theta\epsilon$	$\frac{I}{\pi} \cos \left(-z \frac{\theta\epsilon}{x_1} \right)$	$\theta\epsilon$	$\frac{I}{\pi} \cos \left(-z \frac{\theta\epsilon}{x_1} \right)$
0.0.....	0.5000	2.0.....	0.3252
0.1.....	0.4917	2.1.....	0.3154
0.2.....	0.4834	2.2.....	0.3053
0.3.....	0.4751	2.3.....	0.2951
0.4.....	0.4667	2.4.....	0.2845
0.5.....	0.4583	2.5.....	0.2737
0.6.....	0.4499	2.6.....	0.2626
0.7.....	0.4415	2.7.....	0.2511
0.8.....	0.4332	2.8.....	0.2391
0.9.....	0.4245	2.9.....	0.2267
1.0.....	0.4160	3.0.....	0.2137
1.1.....	0.4073	3.1.....	0.2000
1.2.....	0.3986	3.2.....	0.1854
1.3.....	0.3898	3.3.....	0.1697
1.4.....	0.3809	3.4.....	0.1526
1.5.....	0.3720	3.5.....	0.1334
1.6.....	0.3629	3.6.....	0.1112
1.7.....	0.3537	3.7.....	0.0837
1.8.....	0.3443	3.8.....	0.0410
1.9.....	0.3349	3.8317.....	0.0000

For the rest of the integral between the limits x_i and a , we make use of the expansion

$$J_0^*(adnu) + J_1^*(adnu) = \frac{2}{\pi adnu} + \dots,$$

leaving out of account the fluctuating terms, which are generally very small on integration.

We have to find the integral

$$\frac{2k'}{\pi^2 a} \int_0^{K-u(x_i)} \frac{du}{dn^2 u}, \quad (29)$$

where $u(x_i)$ denotes the value of u corresponding to $adnu = x_i$; in other words, $a\sqrt{k'^2 + k^2\phi_i^2} = x_i$, or $\phi_i = \frac{\sqrt{x_i^2 - a^2\epsilon^2}}{2a}$ nearly. Evidently

$$k^2 \frac{d}{du} \left(\frac{\operatorname{sn} u \operatorname{cn} u}{dn u} \right) = dn^2 u - \frac{k'^2}{dn^2 u};$$

whence

$$\begin{aligned} \int \frac{du}{dn^2 u} &= \frac{1}{k'^2} \int dn^2 u du - \frac{k^2}{k'^2} \frac{\operatorname{sn} u \operatorname{cp} u}{dn u} \\ &= \frac{E\left(\frac{\pi}{2} - \phi_i, k\right)}{k'^2} - \frac{k^2}{k'^2} \frac{\sin \phi_i \cos \phi_i}{\sqrt{1 - k^2 \sin^2 \phi_i}} \end{aligned} \quad (30)$$

following Legendre's notation.

Since

$$E\left(\frac{\pi}{2} - \phi_i, k\right) = \int_{\phi_i}^{\frac{\pi}{2} - \phi_i} \sqrt{1 - k^2 \sin^2 \psi} d\psi = \int_0^{\frac{\pi}{2}} - \int_{\frac{\pi}{2} - \phi_i}^{\frac{\pi}{2}}, \quad (31)$$

we find

$$\begin{aligned} \int_{\frac{\pi}{2} - \phi_i}^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2 \psi} d\psi &= \int_0^{\phi_i} \sqrt{k'^2 + k^2 \phi^2} d\phi \\ &= -\frac{\phi_i}{2} \sqrt{k'^2 + k^2 \phi_i^2} + \frac{k'^2}{2k} \log \left\{ \phi_i + \frac{\sqrt{k'^2 + k^2 \phi_i^2}}{k} \right\} + \frac{k'^2}{2k} \log \frac{k'}{k}. \end{aligned}$$

Further

$$\frac{1}{k'} E\left(\frac{\pi}{2}, k\right) = \frac{1}{k'} + \frac{k'}{2} \left(\log \frac{4}{k'} - \frac{1}{2} \right)$$

Since $k' = \frac{\epsilon}{2}$ nearly, we have

$$\begin{aligned} \frac{E\left(\frac{\pi}{2} - \phi_1, k\right)}{k'} &= \frac{2}{\epsilon} - \frac{x_1 \sqrt{x_1^2 - a^2 \epsilon^2}}{4a^2 \epsilon} + \frac{\epsilon}{4} \left\{ \log 4 - \frac{1}{2} - \log \left(\frac{x_1 + \sqrt{x_1^2 - a^2 \epsilon^2}}{2a} \right) \right\} \\ \frac{k^2 \sin \phi_1 \cos \phi_1}{k' \sqrt{k'^2 + k^2 \phi_1^2}} &= \frac{2 \sqrt{x_1^2 - a^2 \epsilon^2}}{x_1 \epsilon}. \end{aligned}$$

Consequently the expression sought becomes after reduction

$$\begin{aligned} \frac{2}{\pi^2 a} \left(\frac{E\left(\frac{\pi}{2} - \phi_1, k\right)}{k'} - \frac{k^2 \sin \phi_1 \cos \phi_1}{\sqrt{k'^2 + k^2 \phi_1^2}} \right) &= \frac{1}{\pi^2} \left\{ \frac{2a\epsilon}{x_1(x_1 + \sqrt{x_1^2 - a^2 \epsilon^2})} \right. \\ &\quad \left. - \frac{x_1 \sqrt{x_1^2 - a^2 \epsilon^2}}{4\pi a^3 \epsilon} + \frac{\epsilon}{4a} \left(\log 4 - \frac{1}{2} - \log \frac{x_1 + \sqrt{x_1^2 - a^2 \epsilon^2}}{2a} \right) \right\} \end{aligned}$$

nearly. In most cases $a\epsilon$ is very small compared with $4a$, so that the value of the integral turns out to be simply

$$\frac{1}{\pi^2} \cdot \frac{2a\epsilon}{x_1(x_1 + \sqrt{x_1^2 - a^2 \epsilon^2})} \quad (31)$$

which is given in Table VI.

From the quantities given in Tables IV, V, and VI we obtain the value of the integral which enters the expression of the marginal intensity. Table VII gives that part of the integral, which is to be added for internal points, and subtracted for the external, from the peripheral intensity given by (22), which is to be found from Table I. The diffraction phenomena, which appear quite striking, are usually observed near the margin of a luminous disk, when

TABLE VI

$a\epsilon$	$\frac{2\theta}{\pi^2 x_1(x_1 + \sqrt{x_1^2 - a^2\epsilon^2})}$	$a\epsilon$	$\frac{2\theta}{\pi^2 x_1(x_1 + \sqrt{x_1^2 - a^2\epsilon^2})}$
0.0.....	0.0000	2.0.....	0.0149
0.1.....	0.0007	2.1.....	0.0158
0.2.....	0.0014	2.2.....	0.0167
0.3.....	0.0020	2.3.....	0.0176
0.4.....	0.0028	2.4.....	0.0186
0.5.....	0.0035	2.5.....	0.0196
0.6.....	0.0042	2.6.....	0.0207
0.7.....	0.0049	2.7.....	0.0218
0.8.....	0.0056	2.8.....	0.0230
0.9.....	0.0063	2.9.....	0.0242
1.0.....	0.0070	3.0.....	0.0255
1.1.....	0.0078	3.1.....	0.0270
1.2.....	0.0085	3.2.....	0.0285
1.3.....	0.0092	3.3.....	0.0302
1.4.....	0.0100	3.4.....	0.0321
1.5.....	0.0108	3.5.....	0.0344
1.6.....	0.0116	3.6.....	0.0370
1.7.....	0.0124	3.7.....	0.0405
1.8.....	0.0132	3.8.....	0.0465
1.9.....	0.0140	3.8317.....	0.0529

TABLE VII

$$I_2 = \int_0^\pi \left(I - J_0^2[a(1+\nu)\sqrt{1-k^2 \sin^2 \psi}] - J_1^2[a(1+\nu)\sqrt{1-k^2 \sin^2 \psi}] \right) \frac{k' d\psi}{1-k^2 \sin^2 \psi}$$

$a\epsilon$	I_2	$a\epsilon$	I_2
0.1.....	0.0270	2.1.....	0.4008
0.2.....	0.0539	2.2.....	0.4080
0.3.....	0.0806	2.3.....	0.4141
0.4.....	0.1066	2.4.....	0.4194
0.5.....	0.1323	2.5.....	0.4238
0.6.....	0.1571	2.6.....	0.4276
0.7.....	0.1812	2.7.....	0.4308
0.8.....	0.2045	2.8.....	0.4336
0.9.....	0.2268	2.9.....	0.4359
1.0.....	0.2479	3.0.....	0.4378
1.1.....	0.2679	3.1.....	0.4393
1.2.....	0.2868	3.2.....	0.4406
1.3.....	0.3044	3.3.....	0.4419
1.4.....	0.3208	3.4.....	0.4430
1.5.....	0.3357	3.5.....	0.4439
1.6.....	0.3496	3.6.....	0.4450
1.7.....	0.3623	3.7.....	0.4458
1.8.....	0.3736	3.8.....	0.4469
1.9.....	0.3839	3.8317.....	0.4471
2.0.....	0.3933		

another luminous point or disk comes nearly in contact with it, or when a dark disk passes over a luminous disk. We have therefore to lay special stress on the treatment of this particular case. The difficulty hitherto encountered was principally in the evaluation of (27) and (31), and especially of the latter. By the simplification here obtained, the different aspects of the phenomena can be easily studied from the tables of the numerical quantities involved in the problem.

A luminous point and a bright disk.—Having solved the problem in four steps, we can proceed to the discussion of several cases of interest in observations with telescopes.

Suppose a luminous point is situated on the margin of a bright disk. The intensity of the diffracted image due to a point source is given by $\left(\frac{2J_1(r)}{r}\right)^2$, and the lines of equal intensity or the isophotes are concentric circles, whose position is easily given by the tables calculated by Lommel. If we superpose on this system of isophotes those belonging to a bright disk, we can graphically obtain a system of isophotes due to a luminous point and a bright disk. The diagrams are given in Figs. 5a, b, and c; the intensity of the point and that of the disk ($a=50$) at the center is taken to be unity. The actual position of the point and the periphery of the disk are shown by an asterisk and a dotted line respectively, while the isophotes are shown in full lines, the numbers indicating the intensities. The normal distance between the point and the periphery being given by d , the inspection of the figures indicates that with the approach of the point to the disk the place of maximum intensity is gradually shifted toward the disk; when the point comes exactly in contact with the periphery of the disk the luminosity is most intense a short way inside the disk; the consequence is that the point apparently enters the surface of the disk before it is actually covered by coming behind the disk, if it lay farther away.

Two bright disks.—By a procedure similar to the foregoing we can easily draw the isophotes for two bright disks nearly in contact. The isophotes are shown in Figs. 6a and b, which are drawn on the supposition that the intensities for both disks are equal. It

will be easily seen from the course of the isophotes that the contact of two disks is difficult to judge on account of the gradual transition of the isophotes, which depend mainly on the aperture of the telescopic objective. Thus there may be some difference in the

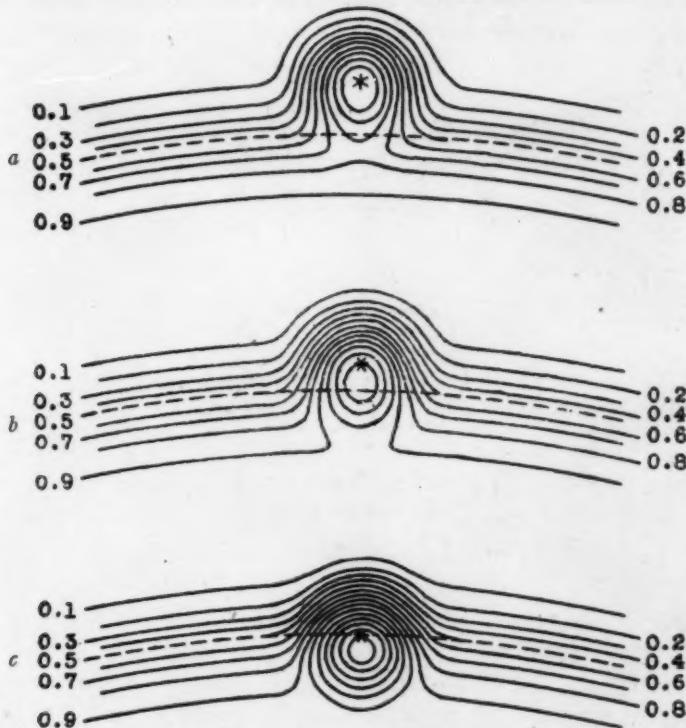


FIG. 5.—Luminous point and bright disk, $a=50$

- a. $d = -2$
- b. $d = -1$
- c. $d = 0$

measurement of the moment of contact of a disk either with a luminous point or with a bright disk according to the instrument of observation.

A dark disk in a bright disk.—The case presenting much interest is the case of a bright disk partly covered by a dark disk; this corresponds in astronomical phenomena to the transit of the inferior planets over the sun's disk.

Constructing the isophotes in the same manner as the preceding, by superposing the intensity with positive sign due to the bright disk on that with negative sign due to the dark disk, we find various transitions of the isophotes, as the dark disk gradually covers a portion of the bright disk. The various stages of progress

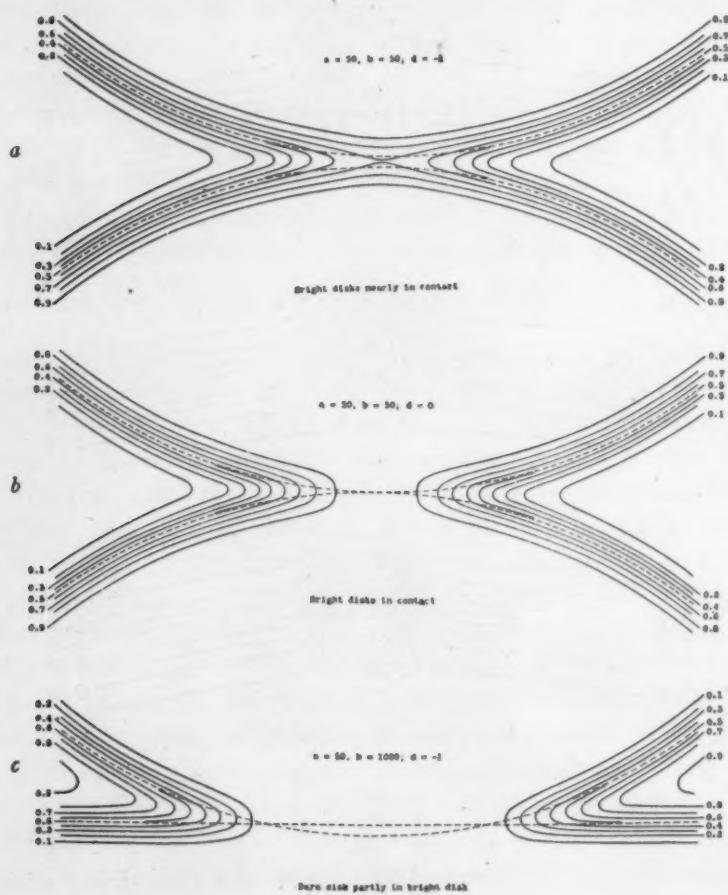


FIG. 6

are shown in Fig. 6c and Fig. 7, the radius of the dark disk being $a = 50$ and that of the bright disk $b = 1000$.

When the dark disk is partially in the bright disk and when they are in contact, the dark surface is apparently connected with the region external to the bright disk. The isophotes show curious

trends, so that the space bounded by them resembles a dark drop. When the shortest distance between the disks $d=0.38$; in the diagram here given, the isophote of intensity 0.1 surrounding the

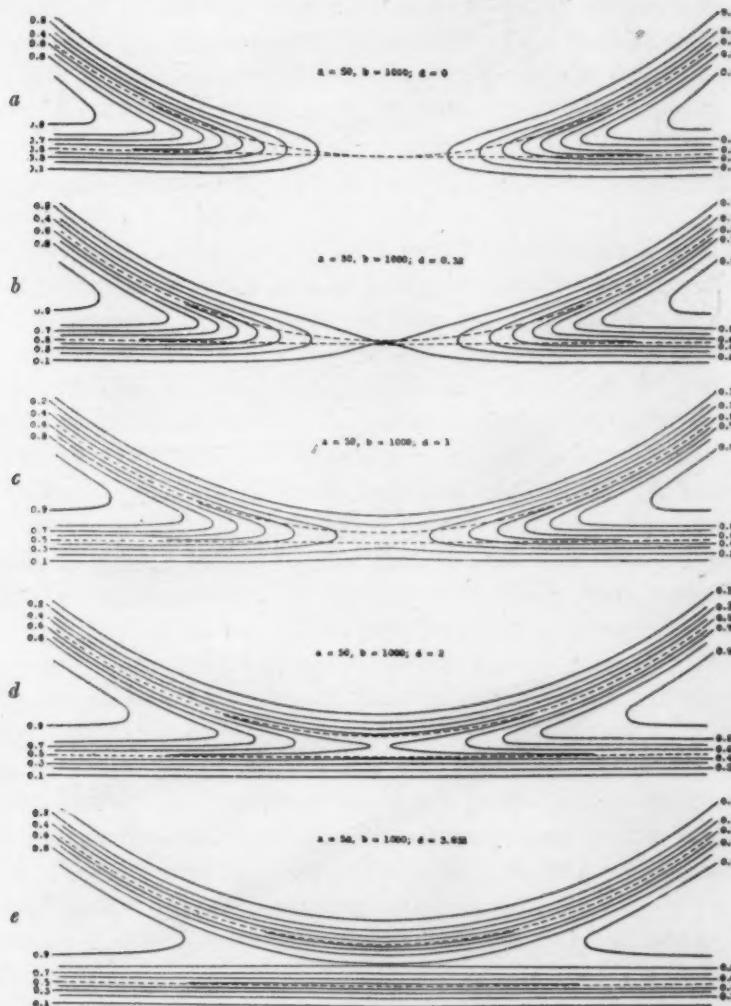


FIG. 7.—Dark disk in bright disk

dark disk comes nearly in contact with that of the same intensity outside the bright disk (Fig. 7b); the curve shows a small protuberance, and the dark ligament which connects the dark space

inside and outside the bright disk is on the verge of disappearing. On passing this curious transient state, the protuberance gradually

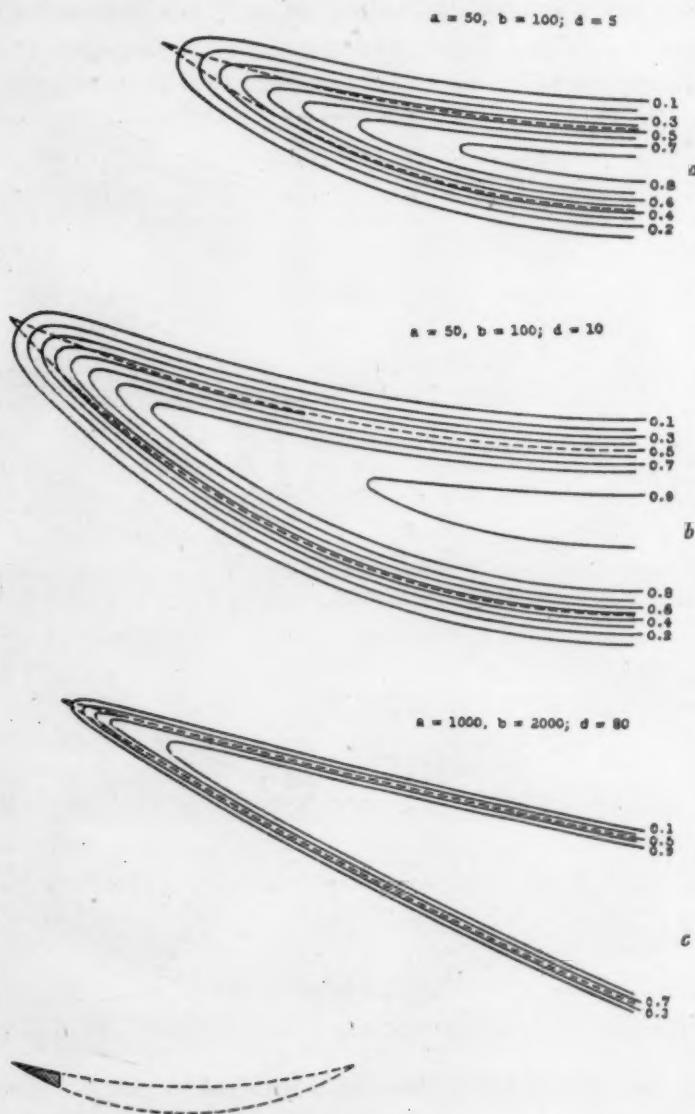


FIG. 8

disappears, and the dark disk assumes a circular shape as it enters deeper into the bright disk, until the deformation in the contour

of both disks becomes indistinct, as a glance at the isophotes in Figs. 7c, d, and e for $d=1, 2, 3.83$, will show. Some of these curves have already been given in my former paper, but as the method of calculating the isophotes was extremely tedious, the transient stages could not be followed closely as here given.

It is needless to remark that when $a=b$, and the radius is made very large, while d is small, the distribution of the intensity will approach that in the image of a long slit. The intensity lacks uniformity toward the edge, as can be easily conceived from the construction of Fig. 7e.

Lune.—A similar process leads to the construction of isophotes in a luminous lune. As will be easily conceivable, the distribution of the isophotes in and about the lune differs with the radii of the bounding circles. When they are small and the lune thin, the isophotes are as shown in Fig. 8a ($a=50, b=100, d=5$), while for double the above-mentioned thickness, they are given in Fig. 8b. For very large radii and corresponding increase in thickness ($a=1000, b=2000, d=80$), only a small portion of the boundary is affected, as shown in Fig. 8c, of which a shaded portion in the annexed figure is given. It will be seen that the effect of diffraction is to round the extremities of the lune, so as to change them into curves having finite curvature. Thus the effect will be strongly felt, when the radii of the bounding circles are small.

Conclusion.—Examples of like nature can be multiplied by considering various other cases, but it will be sufficient to indicate the variation in the image of the luminous surface caused by the diffraction of the observing telescope. In the actual observation, physiological effects such as irradiation may cause further modification of the observed image, but it is outside the scope of the present investigation. What I want to show is the rôle played by the diffraction of the telescopic objective in forming the image of a circular source.

In preparing the present paper my thanks are due to Mr. Sadazo Sakurai, assistant in the Institute for Physical and Chemical Research, for carrying out complicated numerical calculations and for verifying various formulae in the present paper.

ON COMET 1919 b AND ON THE REJECTION OF A COMET'S TAIL

BY E. E. BARNARD

ABSTRACT

Comet 1919 b , which is a return of Brorsen's comet V of 1847, is interesting both from a historical standpoint and because of its tails. During September and October, 1919, it was visible to the naked eye as a dim, hazy star without any tail, with a maximum brightness corresponding to about magnitude 4.5. Twelve photographs were taken. The first ones showed only a slender tail several degrees long, but later the comet became fairly active and about October 20 *discarded its tail* and developed a new one which made an angle of 12° with the old.

Rejection of a comet's tail.—The instances of this phenomenon which have been observed previously are the following: Borrelly's comet in 1903, Morehouse's comet in 1908, and Halley's comet in 1910. The case of *Morehouse's comet* on October 15, 1908, is particularly interesting, for the photographs when combined and viewed with the stereoscope clearly show that the rejection was associated with a cyclonic disturbance. Other features characteristic of the various stages of the phenomenon, the true nature of which the author was the first to recognize, are briefly described.

The first of the two comets discovered in August, 1919, by Metcalf was shown by Leuschner to be a return of Brorsen's comet V of 1847, which was originally discovered by Brorsen at Altona, Germany, on July 20, 1847, and passed perihelion about September 9 of that year.¹ At its present return the comet passed perihelion on October 16, 1919. It seems to belong to the Neptunian family of comets,² of which group Halley's is the best-known member. Photographs of it, therefore, are interesting from a historical standpoint, if from no other.

At the apparition of 1847 it was a rather faint object and apparently did not attain naked-eye visibility. Various orbits were computed from the observations of 1847 (the best of which was one by B. A. Gould), but though they showed the comet to be certainly periodic the periods assigned were discordant and unsatisfactory.

¹ *Astronomische Nachrichten*, 26, 87, 155, 1847.

² In the *Scientific American* for November 1, 1919, Professor H. N. Russell has shown that Neptune could not be responsible (like Jupiter for his comet family) for the grouping of these comets. Though Neptune may not have been responsible for their capture, the term "Neptunian comet family" may still hold through courtesy.

At the present return this object was visible to the naked eye for over a month as a dim hazy star without any tail. The greatest brightness seemed to be at about $4\frac{1}{2}$ magnitude on the Harvard scale. For a while it was above the horizon throughout the night, and later it could be seen both in the evening and in the morning, and later still only in the morning sky just before dawn.

NAKED-EYE VISIBILITY OF THE COMET

Following are some of the notes made while the comet was visible to the naked eye:

1919 Sept. 15, $7^{\text{h}}45^{\text{m}}$ Central Standard Time. Distinctly visible to the naked eye as a hazy spot. Comparisons with 5 Canum Venaticorum made the comet's magnitude 5.4. With the field glass and the image out of focus comparisons with the above-mentioned star made its magnitude 5.0. At $8^{\text{h}}0^{\text{m}}$ to the naked eye the comet was very slightly brighter than the Andromeda Nebula, but very much smaller.

Sept. 19, $8^{\text{h}}0^{\text{m}}$. Just visible to the naked eye.

Oct. 5, $16^{\text{h}}20^{\text{m}}$. Visible to the naked eye as a small hazy spot of light. By comparison with several stars its magnitude was 4.0, but its light was mixed up with that of 93 Leonis, and the magnitude given is probably too bright.

Oct. 6, $16^{\text{h}}40^{\text{m}}$. Visible to naked eye; $4\frac{1}{2}$ magnitude.

Oct. 7, $16^{\text{h}}30^{\text{m}}$. Not visible to the naked eye because of moonlight.

Oct. 12, $16^{\text{h}}30^{\text{m}}$. Too much moonlight to see it with the naked eye. Brightly condensed in the 5-inch guiding telescope but with no trace of tail on the bright sky.

Oct. 16, $16^{\text{h}}30^{\text{m}}$. Not visible to the naked eye on account of moonlight.

Oct. 20, $16^{\text{h}}50^{\text{m}}$. Faintly visible to the naked eye; 5.6 magnitude. In the 5-inch guiding telescope there was some tail.

The magnitudes are on the Harvard scale.

PHOTOGRAPHS OF THE COMET

During most of the time that the comet was present the Bruce telescope was not available for photographing it, but later, especially in the morning sky when near perihelion, photographs were obtained with the 6-inch and 10-inch lenses of this instrument. But the

exposures were short from moonlight and dawn. Bad weather also interfered with the observations during the most important period. Some of the photographs, however, are valuable. They suggest that had better conditions prevailed the results would have been extremely interesting.

The first photographs, in September, showed only a slender tail several degrees long and of no special interest. The later pictures, however, when, unfortunately, short exposures only were possible, are quite interesting and show that the comet finally became fairly active, especially when past perihelion. This was strikingly the case on or about October 22, when the tail was discarded and a new one formed. The table gives a complete list of the photographs which were secured by the writer with the Bruce telescope. The approximate position angles and length of the tail are also given:

Date	C. S. T.	Exposure	Length of Tail	Position Angle
1919 Sept. 21..	16 ^h 27 ^m	○ ^h 7 ^m	3 [°] 4	2 [°]
	7 37	○ 28	2
	16 5	○ 45	3	5
	7 33	○ 47	1	9
	16 20	○ 41	3	328
	6 14	○ 57	7	328
	16 26	○ 41	7	331 $\frac{1}{2}$
	16 33	○ 43	5	315
	16 27	○ 50	8 $\frac{1}{2}$	312
	16 50	○ 33	7	307 $\frac{1}{2}$
	16 51	○ 24	6	307 nearer part, or new tail
	16 50	○ 40	5	302

In some of these plates the tail is very faint toward its end. The rejected part, in the photographs of October 22, makes an angle of 12[°] with the new tail. The nearest point of this drifting tail is 51' from the head. October 23 was cloudy here so that no photographs could be made. This was unfortunate, as material for the motion of the particles of the tail would have undoubtedly been obtained. The photographs of October 20 may show an earlier stage of this separation. On that date the tail proper seemed disconnected from the head. The rearward portion, which

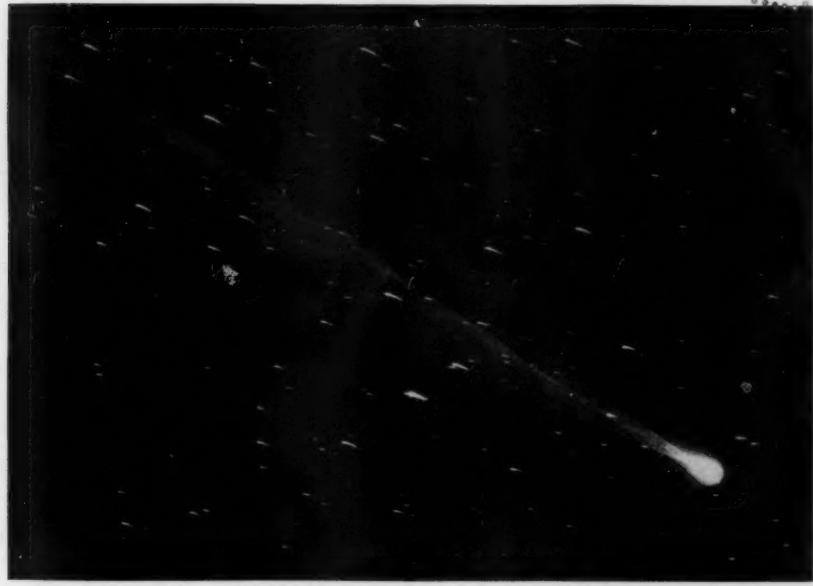
PLATE X

North



a

North



b

COMET 1919b (METCALF-BROSEN)

Scale: 1 cm = 36'

a. 1919, October 5, 16^h20^m C.S.T. Exposure, 0^h41^m
b. 1919, October 6, 16^h14^m C.S.T. Exposure, 0^h57^m

222222
222222
222222
222222
222222

PLATE XI

North

a



b



COMET 1919*b* (METCALF-BRORSEN)

Scale: 1 cm = 36'

- a.* 1919, October 20, 16^h50^m C.S.T. Exposure, 0^h33^m
- b.* 1919, October 22, 16^h51^m C.S.T. Exposure, 0^h24^m

11111111
11111111
11111111
11111111
11111111

was sharply pointed, was $9.6'$ from the head, while a new and widening tail filled the space between it and the head. If these parts of the tail were the same on the two dates, the recession of the particles was at the rate of $21'$ a day. Photographs made elsewhere will probably decide this question. On the photograph of October 20 a brighter condensation about 2° long is shown in the tail $2^\circ 30'$ back from the head. Four of these pictures of the comet, on the dates October 5, 6, 20, and 22, made with the 10-inch lens of the Bruce telescope, are reproduced in Plates X and XI.

This discarding of the entire tail of a comet is not a new feature, though I believe it was unknown previous to the successful introduction of photography to the study of comets. The first known case really occurred in 1903, when on July 24 Borrelly's comet discarded its tail and at once formed a new one. On that date the comet's tail presented a puzzling appearance. It seemed to be split diagonally into two tails. To explain this phenomenon the present writer suggested¹ that the comet had discarded its entire tail and had formed a new one at a slightly different angle and that the old tail was drifting away bodily into space. Though this explanation was somewhat antagonistic to the then received ideas of a comet's tail, it proved to be the true one. It has been amply verified since then by Morehouse's comet on several dates, by Halley's comet on June 6, 1910, and by comet 1919b. In the various cases of this kind a new tail (and sometimes a system of tails) is always sent out at once by the comet, generally in a somewhat different direction from that of the rejected tail.

Perhaps the most interesting case of rejection is that of Morehouse's comet on October 15, 1908, when the nearer end of the receding tail presented a twisted or knotted appearance. Fortunately photographs on that date were secured at a number of observatories, both in this country and in Europe. My own photographs of it cover a period of over seven hours. Thus a fairly full record of these changes was obtained. I have combined for the stereoscope several sets of my own pictures of the comet at that time. The results are very interesting. They clearly show the gradual transformation of the near end of the old tail. At

¹ *Astrophysical Journal*, 18, 213, 1902.

first it was twisted or cyclonic in form, as if it had received some twisting motion when it left the head. It slowly formed into a thickish fragment of a ring, from all parts of which streams of particles swept back to form the old tail, giving it the appearance of part of an open sack, or a partly opened scroll, with irregular sides. Without the aid of the stereoscope one would never have guessed the real form of the tail. It seems that immediately after the separation of the tail from the head a new and slender tail was shot out from the head at a different angle from that of the receding one. In the stereoscope this new tail is seen to pass behind the old one—away from us and toward the background of the stars. It was moving out much faster than the rear portion of the old tail—a peculiarity that seems to be always present in the general process of forming a new tail. This fact was very strongly shown in the case of Borrelly's comet of 1903. It would therefore seem that the rear part of a receding tail is made up of the more sluggish, or larger, particles and is not moving as fast as the other parts of the rejected tail. Measurements of this part will therefore give the minimum velocity of the tail-forming particles.

In *Popular Astronomy*, 17, for November 1909, Plate IX, I have given two photographs of Morehouse's comet on 1908 October 15, that form a stereoscopic view, which beautifully shows the earlier stages of the separation of the tail from the head.

YERKES OBSERVATORY, WILLIAMS BAY, WIS.
January 16, 1920

PRELIMINARY OBSERVATIONS OF THE ZEEMAN EFFECT FOR ELECTRIC FURNACE SPECTRA¹

By ARTHUR S. KING

ABSTRACT

Zeeman effect for furnace spectra of iron and vanadium.—Hitherto Zeeman effect observations have been almost exclusively confined to spark spectra. Because the electric furnace emits or absorbs strongly many lines which are weak or absent in spark spectra, and because it is desirable to compare the effects of a magnetic field on spectra from different sources, the present investigation was undertaken. The furnace tube was placed parallel to the lines of force, in a field varying from 6500 gauss in the center to 9000 gauss near the ends, and it was heated to a maximum temperature of 2200°. To obtain the inverse effect, a carbon plug was placed so as to supply a continuous high-temperature spectrum as a background. Spectrograms were taken with a concave grating giving a scale of about 1.83 Å per mm. Table I contains the observed separations for both the direct and the inverse effects, viewed along the lines of force, for 100 iron furnace lines together with the corresponding separations obtained with the spark as source. Table II contains similar data for the direct effect for 90 vanadium lines. Twenty of the iron lines and eleven of the vanadium lines had not previously been investigated. It was found that the character and magnitude of the separations for the n -components of furnace lines both in emission and in absorption agree closely with those of the corresponding spark lines, reduced to the same field intensity; that is, the effect is independent of the source used. Since these three methods give identical results for each line, they may be used to supplement each other in studying the effect. Among other advantages of the furnace, the non-uniformity of the magnetic field enables the location and temperature of the active centers of absorption or emission for various lines to be determined. Spectrograms are reproduced in Plates XII and XIII.

Zeeman effect for the D-lines of the sodium spectrum.—The author suggests a possible explanation of a variation of the effect with the source of light, which was observed by Woltjer.

On account of the difficulty of maintaining an arc in the magnetic field, investigations of the Zeeman effect have thus far been carried out almost exclusively with the electric spark as the source of light. While the spark is satisfactory for much work of this kind, certain disadvantages make it desirable to supplement the spark experiments with those from a source quite different in character. Two features of the spark spectrum are especially

¹ Contributions from the Mount Wilson Observatory, No. 180.

troublesome. First, many lines characteristic of the low-temperature sources, and important in sun-spot spectra, are faint or absent in the spark, though, on the other hand, the spark emits strongly the enhanced lines and others requiring high excitation. Second, the condensed spark in air, made more disruptive by the magnetic field, is an unfavorable source for obtaining the sharp Zeeman components required for measurements of high precision. A sharpening of the lines by the use of self-induction, which also removes the disturbing air lines, is accompanied by a reduction in the luminosity of the spark.

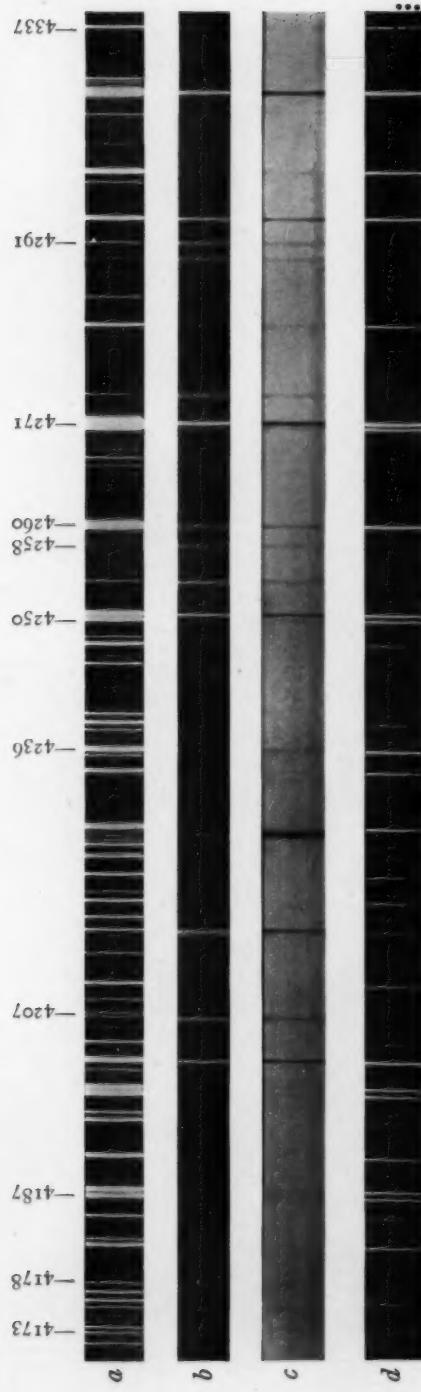
The electric furnace is especially effective in emitting the lines which are weak in the spark. When inclosed in a vacuum chamber, the furnace lines can be made extremely sharp, and, unlike the sources involving an electric discharge, neither the action of the furnace nor the character of its spectrum undergoes material modification when operated *in vacuo*. A third useful feature of the furnace is the unique facilities afforded for the study of the inverse Zeeman effect by the production of absorption spectra when a diaphragm of graphite is placed at the center of the tube. The absorption spectra produced in the regular furnace by this means were described in a recent paper.¹

The obvious disadvantage of the furnace when used with the usual type of electromagnet is its relatively large size, which prevents the use of high magnetic fields. While an equipment designed to overcome this difficulty is being prepared, it seemed desirable, by operating a small furnace between the poles of a Weiss electromagnet, to examine the character of the magnetic separations given by the moderate field available. The possibilities of the furnace for this work could thus be determined and it could be seen whether any decided peculiarities appeared for a source very different from the spark.

This investigation has been limited to the spectra of iron and vanadium, with the addition of a few impurity lines, but the material is sufficient to show clearly the nature of the magnetic separations for the furnace spectrum.

¹ *Mt. Wilson Contr.*, No. 174; *Astrophysical Journal*, 51, 13, 1920.

PLATE XII

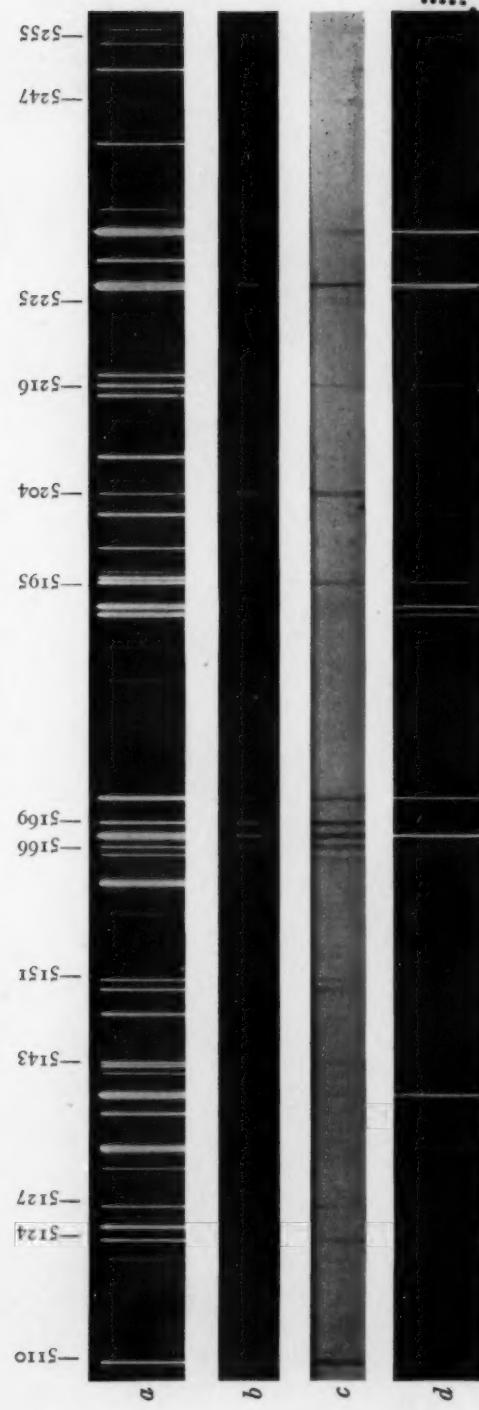


EFFECT OF A MAGNETIC FIELD ON THE ELECTRIC FURNACE SPECTRUM OF IRON

- a, d.* Comparison arc spectra of iron without field
- b.* Emission spectrum of furnace in magnetic field
- c.* Absorption spectrum of furnace in magnetic field

ଶ୍ରୀକୃଷ୍ଣ
ଦେବ
ପାତ୍ରମନ୍ତି
ଅଧ୍ୟାତ୍ମିକ
ପାଠାଳ

PLATE XIII



EFFECT OF A MAGNETIC FIELD ON THE ELECTRIC FURNACE SPECTRUM OF IRON

- a, d. Comparison arc spectra of iron without field
- b. Emission spectrum of furnace in magnetic field
- c. Absorption spectrum of furnace in magnetic field

cccccc
cccccc
cccccc
cccccc
cccccc

EXPERIMENTAL METHOD

A graphite tube 10 cm long, of 6 mm internal and 12.5 mm external diameter, was placed axially between the poles of a large Weiss electromagnet. The latter was provided with pole-pieces 5.1 cm in diameter, one of which was perforated to enable the light from the furnace tube to pass through the hollow core to the spectrograph. The tube was clamped at each end in a graphite bushing 12.5 mm wide held in a brass ring of the same width, the two halves of which were bolted tightly together. This ring was of rectangular cross-section and made hollow to permit of water-cooling. The alternating current, at potentials of 5 to 20 volts, was led to the clamping rings by flexible copper tubes, also water-cooled, from the massive transformer terminals some 2 meters distant. The portion of the furnace tube between the clamps was turned down to a slightly smaller diameter, varying in different experiments, and, since no vacuum chamber was used, it was surrounded by a protecting jacket. Various jacketing methods were tried, but the most satisfactory was to inclose the furnace tube in a thick-walled graphite tube, with an air space of 2 to 3 mm between, and to cool this outer tube with a tubular water-jacket of brass.

A magnetic gap of about 10.5 cm was required to allow space for the furnace. The field in this gap, when the magnet was fully excited, was measured with a bismuth spiral as 6500 gausses at the center, and increased to 9000 gausses 1 cm from the pole, which, however, was considerably beyond the heated portion of the tube. The bearing of this field variation on the observed phenomena of the furnace will be noted later.

The material to be vaporized, either iron filings or powdered vanadium, was placed in the central part of the tube when the emission spectrum was desired, or in front of a graphite plug placed at the center when the absorption spectrum was to be observed. The metal was fused at a moderate temperature before the field was turned on. This, in the case of the iron, which sticks to the graphite when melted, prevented the metal from being drawn out of the tube. The largest current usable with the rather fragile

tube was 800-900 amperes, the temperature attained, judging from the spectrum, being in the neighborhood of 2200° C. The fact that the current passed through the contact blocks for a short distance at right angles to the field caused a vibration which, in spite of fairly rigid clamping in a wooden frame, eventually broke the tube. The limited exposures possible on this account were sufficient to allow photographing with the first order of the 30-foot plane-grating spectrograph and the second order of the 15-foot concave grating, the scales being about 1 mm = 1.83 \AA and 1 mm = 1.86 \AA , respectively.

OBSERVATIONS OF THE ZEEMAN EFFECT

1. *Character of the separations.*—As the spark has hitherto been the only source extensively used in studies of the Zeeman effect, it cannot be assumed without proof that a source so different in its action as the furnace would show no differences in the magnetic behavior of spectrum lines as compared with the spark. The present investigation has therefore had as one of its chief objects a comparison of the furnace and spark material extensive enough to show whether for a given field-strength the number of components and the amount of their separation is the same for the two sources. Since the observations were made entirely along the lines of force, the data collected are for the n -components only. Measurements of the spark separations for iron have been published by the writer,¹ and those for vanadium by Mr. Babcock,² for fields of 16,000 and 20,000 gausses, respectively. These have furnished comparisons of furnace and spark separations for nearly all of the lines except those which are strong in the furnace as compared with the arc or spark. These are chiefly IA lines of the furnace classification and in most cases the measurements given here are the first that have been made for such lines.

A large variety is found in the appearance of the lines emitted by the furnace in the magnetic field, but this is readily explained in every case by the data at hand on the temperature class and the character of the magnetic separations shown for the spark in high

¹ *Papers of the Mt. Wilson Observatory*, 2, Part I, 1912.

² *Mt. Wilson Contr.*, No. 55; *Astrophysical Journal*, 34, 209, 1911.

fields. For lines of triplet separation, the furnace shows a pair of sharp n -components, their sharpness when viewed along the varying field in which the furnace is placed indicating that the particles emitting them are rather definitely localized in the tube. Many cases of blurred components occur, the line sometimes appearing as a diffuse patch. These are found to be either complex lines in which the four or more n -components are not resolved by the low field, or low-temperature lines whose components are self-reversed, or sometimes a combination of both effects. Interesting cases are observed in which a line has the same appearance as a triplet when viewed at right angles to the lines of force, there being a sharp middle component twice as strong as either side component. This is caused by reversals of the two n -components for which the amount of separation and the width of reversal happen to be such that the red side of one reversed component falls exactly upon the violet side of the other. The furnace results, including the absorption spectra to be discussed later, supply ample evidence that reversed magnetic components, when observed along the lines of force, show no peculiarities as compared with unreversed components of the same lines. A certain amount of evidence on this point has appeared in work with the spark, the difficulty of obtaining some of the ultra-violet iron lines with unreversed components having been noted by the writer.¹

Lines such as $\lambda\lambda$ 5124 and 5435 of iron, which show no separation for high fields with the spark, are sharp also in the furnace spectrum. Several lines which give a quartet of n -components in the spark show the same resolution in the furnace, and the iron line λ 5497 is resolved into five n -components in both sources. The field used with the furnace was too low to separate the more complex types, but in every case of lines common to both, the appearance of the furnace line is explained by the character of its separation in the spark.

2. *The inverse effect.*—Experiments on the magnetic separation of absorption lines were carried out for the iron spectrum to a sufficient extent to show what may be expected of the furnace when operated in this way. The absorption lines obtained with

¹ *Papers of the Mt. Wilson Observatory*, 2, Part I, p. 19, 1912.

a plug in the center of the tube and their dependence on temperature class were discussed in a former paper.¹ It was shown there that successive classes of lines appeared as the temperature was raised, so that the production of narrow absorption lines for a given class is a matter of temperature adjustment. With the absorption furnace in the magnetic field, the components follow closely the character of the no-field absorption line, so that a proper temperature must be chosen for the desired set of lines. Proceeding according to this principle, a large part of the iron lines for which the direct Zeeman effect had been observed were obtained with their components in absorption. Complete similarity as to character and number of components was found for the direct and inverse effects. The absorption method is useful for obtaining low-temperature lines with short exposures, this requiring, as for the no-field spectrum, a higher temperature than that needed for the same lines in emission. For the regular work of obtaining measurable lines under the special conditions supplied by the furnace, the direct effect—adjusting the temperature and vapor-density to the character of various groups of lines—will probably be more used than the inverse effect. The absorption method, however, will have its place in many problems, especially in those connected with solar spectra. For the n -components observed along the lines of force, the present results show that the character of the magnetic resolution is the same for the two effects, so that the components in absorption may be studied by this means when it is of advantage to do so.

3. *Magnitude of furnace separations.*—The separations measured for iron and vanadium are recorded in Tables I and II. A field measurement was first made by selecting a number of lines, each of which has a pair of sharp n -components in both furnace and spark. A comparison of the separations gave for the direct effect a strength of 6400 gauss for the field generally used with the furnace, while a value of slightly less than 6500 gauss was given by a bismuth spiral placed midway between the poles. There is thus excellent agreement of furnace and spark separations, provided the furnace components are emitted by vapor near the center

¹ *Mt. Wilson Contr.*, No. 174; *Astrophysical Journal*, 51, 13, 1920.

of the tube. This is probably the case, as the lines compared were as a rule furnace lines of moderate strength which the central vapor should emit most strongly, the strongest low-temperature lines being usually either overexposed or complex. In Tables I and II, therefore, the spark values, so far as they are available for the given lines, are reduced to a field of 6400 for comparison with the furnace separations.

A comparison of the inverse and direct effects for the same lines showed the separation of the absorption components to be consistently larger than in emission, the separations of the best lines being in the ratio of 1 to 0.87, a result repeated on a number of plates for which the same field was used. This indicated a larger field-strength in the part of the tube where the absorbing vapor was effective. Measurements along the pole-gap with the bismuth spiral brought out the interesting fact that at about 25 mm from the center the field increased to the value indicated by the absorption lines. It follows that the absorbing particles were centered about a position at this distance from the plug. The distance probably varies for different sets of lines; the field and dispersion now used are not sufficient to test this point.

Besides explaining the larger separations of the absorption components in the present experiments, this result suggests a method of locating the section of the furnace tube in which any line is emitted or where the vapor absorbs the line most strongly, and from this the corresponding temperature. A knowledge of the variation of field-strength along the axis of the magnet would be required, and also the temperature-gradient along the tube, which can be measured by moving a plug from end to center and recording its temperature at different points. A high precision in the measurement of the magnetic separations should then give valuable information as to the origin of different groups of lines.

4. *Comparison with spark separations.*—The list of iron lines in Table I gives in the second column the temperature-class, in the third the emission separations as measured, in the fourth the absorption separations reduced by the factor required on account of the larger field apparently operating in this case, and in the fifth the

TABLE I
SEPARATION OF IRON LINES IN MAGNETIC FIELD

λ	CLASS	$\Delta\lambda$		
		Emission	Absorption	Spark
3940.885	IA		.186	.184
4100.745	IA	.176	.185	
4147.675	I	.231	.220	.270
4152.176	IA	.114		
4172.748	IA		.116	
4174.917	IA		.116	.150
4177.598	IA		.142	
4206.696	IA		.191	.135
4216.185	I		.188	.183
*4222.225	III	.174		.190
4232.724	IA	.317		
*4235.953	II	.174	.180	.181
4250.134	III	.163	.157	.153
4258.386	IA	.264	.285	
*4260.489	III	.173	.185	.169
*4271.171	III	.161	.161	.158
4282.406	II	.150	.150	.124
4291.472	IA	.233	.250	
*4299.254	II	.172	.161	.162
4315.092	III	.216	.216	.207
4337.052	II	.097		.106
4347.239	IA	.182		
4352.740	II	.178		.166
*4375.934	I		.180	.170
4389.251	IA		.166	
*4427.313	I		.193	.172
4430.622	II	.304		.288
4435.154	IA	.174	.176	
4442.349	III	.193		.194
4447.727	II	.244		.234†
4459.128	II	.200		.180
*4461.658	I		.182	.174
4466.557	I	.180	.172	.137
4482.176	I		.186	.160
4489.744	IA	.178	.190	
4494.571	II	.134	.128	.121
4528.624	II	.147		.143
4531.155	I	.147		.159
4592.658	I	.152		.166
4602.946	I	.224		.226
*4878.226	III	.455		.437
4939.689	I	.221		.232
4957.612	III	.181		.200
4994.133	I	.234	.211	.232
*5012.073	I	.212	.222	.215
5041.079	I	.232	.228	.199
*5041.763	III	.223	.221	.200
*5051.643	I	.209	.206	.208
5079.742	I	.337	.321	.295
5083.344	I	.196	.200	.190
5110.414	I	.238	.260	.216

TABLE I—Continued

λ	CLASS	$\Delta\lambda$		
		Emission	Absorption	Spark
5127.364.....	I	.257	.254	.270
5142.934.....	I	.238	.231	.231
5150.845.....	I	.246	.256	.252
5166.288.....	IA	.293	.301
5167.492.....	II	.173	.181	.185
5168.904.....	IA	.260	.266
*5171.601.....	II	.209	.214	.208
*5194.950.....	I	.177	.175	.183
*5204.585.....	IA	.293	.284
5216.277.....	II	.094	.104	.122
5225.533.....	IA	{ .506 .260
5227.187.....	II	.169	.171	.163
5232.954.....	III204	.223
5247.052.....	IA	.349	.341
5250.212.....	IA	.513	.506
5254.950.....	IA	.400	.381
*5270.357.....	II	.120	.117	.120
5328.539.....	II	.217	.217	.195
5397.135.....	I260	.252
5429.701.....	I240	.243
5497.521.....	I	{ .568†† .288	{ .556†† .277
5501.471.....	I	.368400
5506.785.....	I	.404410
5615.663.....	IV	.239234
5658.836.....	IV	.280290
5701.555.....	III	.224243
*6137.704.....	III	.254262
6173.346.....	III	.605636
*6191.568.....	II	.191216
6213.440.....	III	.456480§
6219.290.....	III	.396396
*6230.732.....	III	.209307
*6252.567.....	III	.213233
*6254.266.....	III	.355381
6265.145.....	III	.392388
6280.622.....	IA	.359
6301.524.....	IV	.404425
6322.690.....	III	.389386
6335.341.....	III	.288266
6358.684.....	IA	.447208
6393.609.....	III	.213237
*6421.361.....	III	.389397
6430.859.....	III	.310
*6494.993.....	II	.258273
6546.251.....	III	.209234
*6592.928.....	III	.262280
6663.452.....	IV	.411435
*6678.001.....	III	.292315

† Mean of two pairs.

†† Five n -components. Measurements for outer and inner pairs.

§ Mean of two pairs.

spark values reduced to the furnace field. Table II, for vanadium, is similar, except that the absorption method was not used for this element.

These lists include the lines for which more or less sharply defined doublets could be measured. When the furnace components were not sharp, those of the spark showed a pronounced widening or a resolution into an inner and an outer pair. Many strong furnace lines, whose complex character allowed no resolution in this field, are omitted.

A general close agreement of the furnace and spark separations in the tables indicates that the action of the field is the same for the two sources. In Table I the iron lines which in the spark show two sharp n -components and yield measurements of high weight are marked with an asterisk. The agreement for these lines is good. For the other lines, the furnace and spark values often agree closely, and when different show no systematic variation. The mean separations of 67 iron doublets measured for both furnace and spark are 0.2516 and 0.2537 respectively for the two sources. A similar comparison of 79 vanadium doublets gives means of 0.3083 and 0.3100. In the few cases where the discrepancy is considerable, the line is invariably found to be especially difficult to measure in one or both sources, usually owing to a shading of the complex n -components toward one side or the other. The vanadium lines of Table II are usually less difficult to measure than those of iron, the only notable discrepancies being for lines of complex structure.

This observational material is presented in some detail on account of the difficulty in handling magnetic-field data if the character or magnitude of the separations depends upon the nature of the source. No indication of such a dependence in the iron or vanadium spectrum has appeared. The field possible with the present form of furnace has not enabled me to check the observations of Woltjer¹ on the components of the D lines of sodium. He examined these lines as emitted by a vacuum tube and as given in absorption by the oxy-acetylene flame, and found that while the spacing remained the same, the relative intensities of the five

¹ Amsterdam Dissertation, 1914; (Abstract) *Physikalische Zeitschrift*, 15, 918, 1914.

TABLE II
SEPARATIONS OF VANADIUM LINES IN MAGNETIC FIELD

λ (EXNER AND HASCHEK)	CLASS	$\Delta\lambda$	
		Furnace	Spark
5176.95	III	.178	.177
5193.20	III	.230	.245
5193.84	III	.197	.165
5195.65	III	.245	.204
5213.82	III	.208
5216.76	IV	.279	.260
5225.90	III	.256	.260
5234.20	III	.156	.153
5241.03	III	.188	.184
5353.56	III	.198
5402.17	III	.174	.175
5415.47	III	.197	.193
5487.42	III	.219	.237
5488.10	III	.182	.217
5517.40	IIIA	.342
5542.91	IIIA	.182
5546.13	IIIA	.357	.361
5547.26	II	.342	.344
5574.20	IIIA	.208
5584.77	I	.290	.299
5586.21	III	.200	.214
5592.63	I	.290	.300
5601.60	III	.208	.216
5604.41	IV	.204
5605.18	II	.327	.362
5627.85	I	.271	.288
5646.29	II	.342	.360
5657.70	II	.290	.309
5668.55	II	.297	.305
5683.37	III	.226	.232
5725.80	III	.215	.219
5733.29	IV	.193	.191
5734.21	III	.189	.199
5737.25	II	.234	.256
5743.66	II	.260	.273
5747.92	III	.200	.228
5749.05	III	.222	.185
5786.43	III	.230	.210
5807.38	III	.197	.180
5826.83	III	.200
5850.57	IIIA	.301	.325
5979.11	IIIA	.241	.229
5981.02	IIIA	.238	.227
6002.89	IIA	.416	.457
6008.90	IIIA	.331	.280
6018.16	IIIA	.301
6025.64	IIIA	.222	.209
6039.95	I	.332	.331
6087.70	IIIA	.312
6090.45	I	.272	.286
6111.90	II	.294	.293

TABLE II—Continued

λ (EXNER AND HASCHEK)	CLASS	$\Delta\lambda$	
		Furnace	Spark
6119.70.....	I	.239	.240
6150.32.....	I	.280	.282
6170.55.....	IA	.286	.269
6189.55.....	IIA	.244	.240
6199.40.....	I	.355	.351
6214.04.....	I	.355	.359
6216.52.....	I	.337	.352
6224.70.....	I	.356	.357
6230.92.....	I	.336	.339
6233.31.....	IA	.356	.349
6240.30.....	IIA	.357	.364
6245.35.....	IIA	.359	.359
6252.02.....	I	.376	.378
6257.03.....	IIA	.394	.396
6258.73.....	IIA	.797	.785
6261.39.....	IIA	{ .743 .181	{ .770 .190
6266.49.....	IIA	.508	.563
6268.98.....	IIA	.458	.409
6274.80.....	I	{ .587 .303	{ .607 .308
6285.32.....	I	.379	.385
6293.02.....	I	.380	.371
6296.69.....	I	.368	.377
6327.00.....	III	.268	.278
6339.23.....	III	.257	.260
6349.61.....	III	.238	.234
6357.47.....	III	.186	.196
6358.99.....	III	.297	.278
6430.68.....	III	.370	.330
6433.37.....	IIIA	.279	.209
6452.55.....	IIA	.394	.384
6488.22.....	IV	.249	.292
6504.38.....	IIA	.348	{ .582 .404
6531.65.....	II	.433	.421
6543.71.....	IIIA	.456
6566.10.....	IIIA	.732	.708
6606.22.....	IIIA	.388	.368
6608.06.....	IIIA	.264
6625.10.....	IIIA	.407	.375

n-components of D₂ were quite different in the two cases. This difference Woltjer provisionally ascribes to temperature. The results of the writer's recent study of absorption spectra¹ with the furnace may, however, have some bearing on this case. Instances were there noted of the weakening of spectral lines through a

¹ *Loc. cit.*

partial balancing of emission and absorption. The effect of large differences in furnace temperatures on the magnetic separations has not been fully tested, owing to the long exposures required at low temperature, but a temperature difference of at least 400° produced no perceptible changes other than those clearly due to reversals at the higher stage.

5. *Magnetic characteristics of Class IA lines.*—This class, especially in the iron spectrum, presents interesting features. It is an important group of lines for which the magnetic data must depend largely on furnace observations, since these lines are very difficult in the spark when they can be measured at all. In number they include 23 per cent of the lines given in Table I. The sharpness of the components in a large majority of cases indicates a greater prevalence of the triplet type than is found among the lines of other classes. In the blue the average separation of IA lines is about the same as for others, but in the green it is nearly twice as great. Simple and often large separations appear to be characteristic of these lines, which are strong in the low-temperature furnace, and very faint in the arc and spark.

Other large groups of lines, for which the furnace is not so necessary as for those of Class IA, are yet relatively strong in the furnace and their Zeeman components appear with less difficulty than with the spark. For a large proportion of the lines in a spectrum the usefulness of the furnace in observations supplementary to those of the spark seems thus to be assured.

DESCRIPTION OF THE PLATES

Plates XII and XIII illustrate the results obtained for both emission and absorption spectra. Arc spectra with long and short exposures, respectively, are given above and below for each section, with the furnace spectra between. The plates selected for reproduction favor the weaker furnace lines so that many of the strong lines are overexposed, some of these showing in the negative the apparent triplet structure resulting from the reversal of both n -components. It will be noted that numerous lines, well defined in the furnace, are scarcely visible in the lower arc exposure, which is a normal one for the stronger arc lines. The very strong arc

spectrum above is necessary to show these IA lines for which the furnace is especially effective.

SUMMARY

1. The action of a magnetic field on the furnace spectrum has been examined both for the direct and inverse effects.
2. The character and magnitude of the separations for the n -components of furnace lines in emission agree closely with those of the spark in a corresponding field. Self-reversed components are frequent and follow the character of the no-field line.
3. The use of a plug in the furnace tube gives the Zeeman components in absorption. The effects agree with those obtained for emission, except that uniformly larger separations for absorption components indicate that the absorption particles are localized in a part of the tube having a higher field-strength.
4. The large class of lines which are relatively strong in the furnace spectrum makes this source especially useful in supplementing the results for the spark in the magnetic field. For the more pronounced type of these lines, not usually obtainable with the spark, the furnace provides an effective method of increasing the data on magnetic separations.

The writer is indebted to Miss Brayton, of the Computing Division, for assistance in measuring a part of the spectrograms.

MOUNT WILSON OBSERVATORY
December 1919

THE SPECTRUM OF NOVA OPHIUCHI 1919¹

BY WALTER S. ADAMS AND CORA G. BURWELL

ABSTRACT

Spectrum of Nova Ophiuchi.—The paper reports in detail measurements of a spectrogram obtained with the 100-inch reflector on November 2. Although the star had reached its maximum brightness over six weeks before, the general features of the spectrum are characteristic of novae at a comparatively early stage of development. The strong continuous background is crossed by bright bands, with absorption lines forming their violet edges, which correspond mainly to hydrogen and calcium lines and to the enhanced lines of iron and titanium. The middle of each band is displaced about 1 Å to the red, while the absorption lines are displaced from 4.5 to 5.5 Å to the violet. The shifts for the various lines are proportional to the wavelength and are about half the width of the associated bands. The *radial velocity* indicated by the calcium absorption lines is about +6 km.

Spectra of novae; a simple interpretation of the observed displacements.—The fact that for Nova Persei, Nova Geminorum No. 2, Nova Aurigae and Nova Ophiuchi the displacement of the absorption lines is in each case about twice the width of the corresponding bands, suggests that both phenomena are mainly Doppler effects, and lends some support to the *hypothesis* of a shell of gas moving rapidly out from each star.

The spectrum of the nova in Ophiuchus discovered by Miss Mackie on photographs made at the Harvard College Observatory² was observed at Mount Wilson on the nights of November 1 and 2. The magnitude of the star was estimated at about 8.0, and for this reason and because of its low altitude it was necessary to use a spectrograph of small dispersion. For the first photograph the 60-inch reflector and the Cassegrain spectrograph with one prism and a camera of 18 cm focal length were employed; the second spectrogram was made with the 100-inch reflector and a spectrograph containing one prism and a camera with a focal length of 46 cm. The latter negative obtained with an exposure time of 110 minutes was most satisfactory, and the results given in this communication are based upon its measurement.

¹ Contributions from the Mount Wilson Observatory, No. 179.

² Harvard College Observatory Bulletin, No. 696.

The general features of the spectrum are those characteristic of novae at a comparatively early stage of development. This is in itself remarkable, since the maximum of light of the star apparently occurred about September 13, over six weeks previous to these observations. The continuous spectrum is strong and a large number of fairly well-defined absorption lines are present. In many cases these are associated with bright bands upon the side of longer wave-length. These bands, as has been found in other novae, are due to hydrogen, calcium, and the more prominent enhanced lines of iron, titanium, and other elements. It is probable that all the absorption lines are accompanied by bright bands, but unless the latter are rather strong it is not possible to distinguish them with certainty against the continuous background. There is little difficulty in identifying essentially all of the absorption lines with the enhanced or the stronger arc lines of various elements. The lines present are nearly identical with those found in the spectrum of Nova Aquilae No. 3 a few days after maximum of light, but their displacement is only about -4.7 angstroms at $\lambda 4500$, against a displacement of -23 angstroms in Nova Aquilae.

RADIAL VELOCITY

A bright band is present in the position of the calcium line K and another band is made up of calcium H and the hydrogen line H ϵ . Across these bands are dark lines due to calcium, the measurement of which affords a means of determining the radial velocity of the star. The lines are somewhat stronger than in most novae, but the measurements are subject to considerable uncertainty because of the inferior focus in this part of the spectrum. The mean of the determinations by two observers, after correction for the earth's motion, is $+6$ km.

ABSORPTION LINES

The principal dark lines measured in the spectrum are given in Table I. Successive columns show the wave-length, as taken from Rowland's table, of the line with which the stellar line is identified, the element to which it is due, the wave-length in Nova Ophiuchi, the displacement, and the corresponding displacement

TABLE I

SUN	ELEMENT	NOVA OPHIUCHI	DISPLACEMENT	
			Nova Oph.	Nova Aquilae
3900.68.....	Enh. Ti	3895.79	A -4.9
3913.61.....	Enh. Ti	3909.08	4.5
3933.82.....	Ca	3928.94	4.9	-19.5
3970.25.....	H ϵ	3965.17	5.1	20.9
4012.54.....	Enh. Ti	4007.94	4.60	20.45
4028.50.....	Enh. Ti	4024.24	4.26	19.28
4101.90.....	H δ	4097.57	4.33	20.54
4163.82.....	Enh. Ti	4159.33	4.49	20.68
4172.84.....	Enh. Ti, Fe	4168.29	4.55	20.41
4179.02.....	Enh. Fe	4174.01	5.01	20.86
4184.47.....	Enh. Ti	4180.03	4.44	20.59
4226.90.....	Ca	4222.38	4.52	20.96
4233.33.....	Enh. Fe	4228.78	4.55	21.44
4247.00.....	Enh. Sc	4242.52	4.48	21.10
4290.38.....	Enh. Ti	4285.74	4.64	21.60
4294.20.....	Enh. Ti	4289.61	4.59	21.58
4296.74.....	Enh. Fe	4292.31	4.43	22.09
4300.21.....	Enh. Ti	4295.59	4.62	21.61
4308.08.....	Fe, Enh. Ti	4303.30	4.78	21.48
4314.09.....	Enh. Ti	4309.68	4.41	21.31
4321.12.....	Enh. Ti	4315.92	5.20	21.62
4325.94.....	Fe	4321.37	4.57	21.98
4340.63.....	H γ	4336.03	4.60	21.68
4352.01.....	Cr, Mg.	4347.21	4.80	21.69
		4363.83		
4375.10.....		4370.28	4.82	21.84
4385.55.....	Enh. Fe.	4380.57	4.98	20.14
4395.20.....	Enh. Ti	4399.49	4.71	21.98
4399.94.....	Enh. Ti	4395.66	4.28	21.50
		4401.51		
4417.43.....	Enh. Fe, Ti	4412.33	5.10	21.20
4443.98.....	Enh. Ti	4439.22	4.76	22.12
4468.66.....	Enh. Ti	4463.89	4.77	22.27
4489.06.....	Enh. Ti, Fe	4484.09	4.97	22.47
4501.44.....	Enh. Ti	4496.56	4.88	22.36
4508.46.....	Enh. Fe	4503.36	5.10	22.45
4515.51.....	Enh. Fe	4510.13	5.38	22.39
4520.40.....	Enh. Fe	4515.48	4.92	22.82
4522.80.....	Enh. Fe	4517.94	4.86	22.35
4528.80.....	Fe	4523.70	5.10
4534.14.....	Enh. Ti	4529.44	4.70	22.50
4549.77.....	Enh. Fe, Ti	4544.57	5.20	22.79
4563.94.....	Enh. Ti	4558.92	5.02	22.65
4572.16.....	Enh. Ti	4567.16	5.00	22.39
4576.51.....	Enh. Fe	4571.98	4.53
4584.02.....	Enh. Fe	4579.06	4.96	23.15
4805.29.....	Enh. Ti	4800.58	4.71	23.86
4861.53.....	H β	4855.88	5.65
4924.11.....	Enh. Fe	4918.44	5.67	24.86
5018.63.....	Enh. Fe	5011.6	7.0	24.97

in the spectrum of Nova Aquilae on June 11, 1918, the date on which the spectrum was most nearly comparable with that of Nova Ophiuchi. The wave-lengths at the beginning and end of the table are less accurate than the others. Corrections have been applied for radial velocity.

The well-known proportionality between displacement and wave-length characteristic of the spectra of novae is marked in these results. Thus if we form means for each 200 angstrom units beginning at $\lambda 4000$ we find the following displacements:

$$\lambda 4120 -4.52 \quad \lambda 4320 -4.65 \quad \lambda 4520 -4.95 \quad \lambda 4900 -5.76$$

These values are satisfied by the equation $\Delta\lambda = 0.00110 \lambda$ with the following residuals, weights being assigned according to the number of lines:

$$\lambda 4120 \text{ o.oo} \quad \lambda 4320 \text{ -o.09} \quad \lambda 4520 \text{ -o.or} \quad \lambda 4900 \text{ +o.38}$$

The ratio of the displacement of the absorption lines in Nova Ophiuchi to that in Nova Aquilae as obtained from Table I is 1 to 4.53. A similar comparison of the displacements in Nova Aurigae, Nova Persei, Nova Geminorum No. 2, and Nova Aquilae, was made by Adams,¹ and attention was called to the remarkably close agreement of the ratios with the integer numbers 1, 2, and 3, the largest value belonging to the more refrangible component of the hydrogen lines. The corresponding value in the case of Nova Ophiuchi should apparently be 4, and the departure from the whole number is too large to come within the limits of error of the determination. Although the relationship found for the four earlier novae may well be regarded as a coincidence, attention should be called to the fact that in the case of at least one, Nova Aquilae, the displacement varied with the interval after maximum of light. Accordingly, results obtained at the same relative light-phase should be used, and no strict comparison can be made between the displacements found in the case of Nova Ophiuchi six weeks after apparent maximum and those in Nova Aquilae only three days after maximum.

One other possible effect may be referred to. As will appear later, the bright bands in Nova Ophiuchi are displaced about

¹ *Proceedings of the National Academy of Sciences*, 4, 355, 1918.

one angstrom toward the red, and those in Nova Aquilae the same amount toward the violet. If these displacements are in any way related to those of the absorption lines, allowance should be made for the difference.

BRIGHT BANDS

A list of the bright bands measured in the spectrum is given in Table II. The values for the first few bands are necessarily uncertain.

TABLE II

Sun	Element	Center of Band	Width in A	Displacement in A
3889.2.....	H ζ	3890	+1
3933.8.....	Ca	3935	7	+1
3970.2.....	H ϵ	3971	8	+1
4028.5.....	Enh. Ti	4029.7	7.3	+1.2
4101.9.....	H δ	4102.9	7.6	+1.0
4233.3.....	Enh. Fe	4233.8	8.0	+0.5
4340.6.....	H γ	4341.7	8.8	+1.1
4417.4.....	Enh. Fe, Ti	4418.5	8.6	+1.1
4444.0.....	Enh. Ti	4444.9	9.1	+0.9
4468.7.....	Enh. Ti	4469.6	8.7	+0.9
4481.4.....	Enh. Mg.	4482.1	+0.7
		4513		
4522.8.....	Enh. Fe	4523.8	8.9	+1.0
4584.0.....	Enh. Fe	4585.7	10.7	+1.7
		4933		
4861.5.....	H β	4863.2	9.2	+1.7
4924.1.....	Enh. Fe	4925.7	10.1	+1.6
5018.6.....	Enh. Fe	5020	11	+1

It is clear from these results that there is a displacement of the center of these bands of the order of one angstrom unit to the red. It also seems probable that both the displacement and the width of the bands increase in proportion to the wave-length. This result is well known in the case of previous novae. The following summary shows the values of the width and displacement at $\lambda 4500$ for the four principal stars of this character:

	Width of Bands in A	Displacement in A
Nova Persei	48	+1.0
Nova Geminorum	25	+1.2
Nova Aquilae.....	49	-1.0
Nova Ophiuchi.....	9	+1.1

The displacement to the violet in the case of Nova Aquilae is of especial interest. It is well established from the observations of Harper at Ottawa¹ and from the Mount Wilson measurements.

A comparison of the widths of the bands in these stars shows that their ratio is closely the same as that between the displacements of the absorption lines, and that in each case the width is very nearly twice that of the corresponding displacement. The absorption lines, therefore, may be regarded as marking definitely the violet edges of the bright bands, and the conclusion is obvious that the cause which produces the displacements of the dark lines must be mainly responsible for the widening of the bright bands. This would favor the view that the Doppler effect is the principal agent involved, since in such laboratory investigations as that of the spectrum of the spark in liquids and under high pressures, in which a slight degree of resemblance to the spectra of novae has been attained, the dark lines and the bright bands have been very differently affected. The hypothesis of a shell of gas moving rapidly outward from the star may, accordingly, be regarded as receiving some slight degree of support from these results.

In conclusion the suggestion may be made that the displacements of the absorption lines and the widths of the bright bands are to some extent an indication of the disturbances present in the star, and so perhaps form a rough measure of its absolute magnitude. From this point of view Nova Ophiuchi would be of especial interest as being intrinsically the faintest of the last five prominent novae.

MOUNT WILSON OBSERVATORY

December 15, 1919

¹ *Journal of the Royal Astronomical Society of Canada*, 12, 494, 1918.

REVIEWS

Theorie der Strahlung und der Quanten. Von ARTHUR MARCH.
Leipzig: Barth, 1919. Pp. 182, figs. 36. 12 marks.

This book seems designed to be a general introduction to the study of various aspects of the theory of quanta, with chief attention to the mathematical developments. Experimental data are referred to in the briefest way sufficient to show the physical bearing of the theories. The topics chosen are in the main those whose investigation may be called somewhat mature. These are perhaps the reasons for omitting consideration of photo-electric phenomena and the quantum relations of cathode particles and X-rays.

About half of the space is given to the classical laws of thermal radiation and Planck's successive deductions of his formula for the spectral distribution of energy. This part is similar in content to Planck's lectures and the order of thought is naturally much the same, even to methods of proof, though more concise. There is a brief chapter on the quantum hypothesis in relation to the statistical theory of entropy, and one on the Einstein and Debye theories of specific heat.

The long fourth chapter is devoted to the relation of quantum theory to spectroscopy and resulting speculations on the structure of atoms. It presents in outline Bohr's theory of the Balmer series and the analogous Moseley X-ray spectra, together with the relativity corrections and Sommerfeld's theory of the details of structure of individual lines. There is no reference to the possibility of a modified theory of the Zeeman effect, but the Stark effect as explained by Epstein and Schwarzschild is treated at some length.

The quantum theory raises many troublesome questions because of its dubious relation to older theories, whose range of success is wide and which as yet it can claim to supplant only in a limited and ill-defined region. Through ignoring these questions and in other ways the present work has unfortunately at times the air of special pleading, but if read with due care it will give a very fair and vivid idea of the remarkable suggestiveness and positive achievements of the new theory.

A. C. LUNN

Advanced Lecture Notes on Light. By J. R. ECCLES. Cambridge: The University Press, 1919. American agents, G. P. Putnam's Son's. Pp. 141. \$2.50.

This is a handsome book of size 21×25 cm, printed in large type, on one side of the page, with margins that give no suggestions of economy of paper. There are no illustrations. The Preface states that the book was first printed for private circulation, we must infer among the science masters of secondary schools. The title conveys an entirely erroneous impression for American teachers, because "advanced" merely distinguishes this from an earlier and still more elementary work by its author. The book covers what would be expected of students in a good high school or preparatory school in the United States, or for a beginner's course in optics in college. Only arithmetic and algebra are involved, except for the occasional use of a trigonometric function, as in the case of the law of refraction. The titles of the sections are as follows: "Rainbows," "Magnifying Power," "Chromatic Aberration," "Spherical Aberration," "Wave Theory of Light," "Interference," "Diffraction," "Polarization of Light." The mode of derivation and presentation of formulae is conventional and the book seems to be free from misprints. There is no index.

F.

NOTE

We are informed that our esteemed collaborator Professor Heinrich Kayser has recently resigned his position as professor of physics and director of the laboratory at the University of Bonn, on account of poor health and advancing years. He intends, however, to continue some work in spectroscopy. A second and wholly re-written edition of the first volume of his monumental work *Handbuch der Spectroscopie* was ready for publication at the beginning of the war, but it is now uncertain when the printing can be resumed, if ever.

The high reputation of the laboratory at Bonn as a center of spectroscopic research will be maintained by the appointment of Professor F. Paschen, of Tübingen, as successor to Professor Kayser.